Spatial Consumption Risk Sharing

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Abstract

This paper examines how frictions in bilateral economic linkages shape the consumption pattern across economies. Using state-level data from the US, we find that the degree of bilateral consumption risk sharing across economies decreases in geographic distance. To explain this novel fact, we develop a DSGE model that incorporates trade, migration, and finance as channels of risk sharing which are subject to frictions that covary with distance. Calibrated to the US data, the model not only enables us to quantify the magnitude of the frictions in each channel, but also allows us to examine the interplay among the channels and disentangle their effects on the level, volatility, and comovement of consumption across states. Counterfactual analyses based on the model shed light on the design of macroeconomic policies that aim to reduce cross-region consumption disparity.

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1 Introduction

Consumption risk sharing allows agents from different economies to yield welfare gains by reducing consumption fluctuations caused by idiosyncratic output shocks. However, there exist frictions in economic exchanges across regions that impede consumption from being smoothed across space and time. This paper explores the patterns and determinants of consumption risk sharing by exploiting the variations in bilateral economic linkages shaped by geography.

In the macroeconomic literature, what drives imperfect consumption correlations across economies remains to be a central question of interest since the phenomenon attests to the failure of complete markets. For example, Obstfeld and Rogoff (2000) consider the low cross-country consumption comovement as one of the major puzzles in international macroeconomics. Besides trade costs in the goods market discussed by these authors, migration costs in the labor market, as well as asset transaction costs in the financial market, potentially affect risk sharing since they pose barriers for economic resources to be freely mobile across economies in the presence of local shocks. Contrary to most existing literature that examines the influence of one friction, this paper extends the workhorse open economy real business cycle (RBC) model developed by Backus et al. (1992) (BKK hereafter) into a unified theoretical framework with trade, migration, and finance as channels of risk sharing. This framework enables us to quantify the magnitude and disentangle the effects of frictions in different channels.

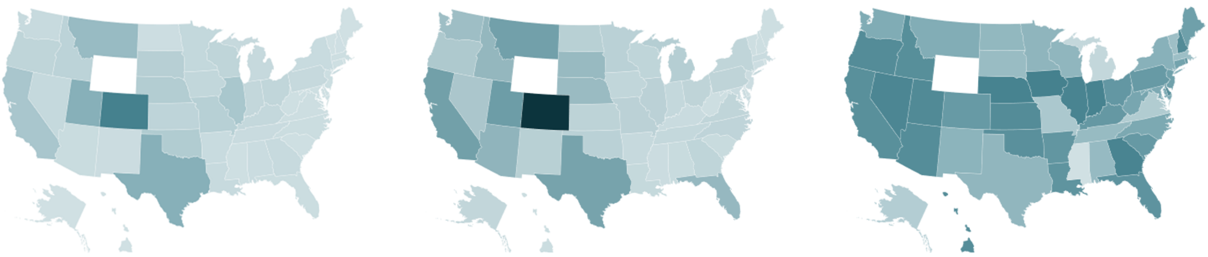
Another distinct feature of this paper is that we add a geographic dimension to our macroeconomic analysis. One similarity of the three channels of risk sharing lies in the fact that economic linkages in these channels covary with geographic distance between a pair of economies, as is documented in the literature as the gravity model of trade, finance, and migration.[1] Since these channels are important drivers for cross-economy synchronization, bilateral consumption comovement is also expected to exhibit similar geographic characteristics. To exemplify such patterns, we plot the bilateral economic ties between Wyoming and other states in the US in figure 1 and confirm that ties are generally stronger for neighboring states.[2] To capture these spatial characteristics, we

[1]For example, Anderson and Van Wincoop (2003) develop a theory-grounded econometric model to characterize bilateral trade flows across countries. Portes and Rey (2005) document that bilateral equity flows decrease with distance between country-pairs. Lewer and Van den Berg (2008) develop and test a gravity model of immigration among OECD countries.

[2]Detailed data description can be found in Appendix B. Cross-state trade data are sourced from the CFS, migration data are from the IRS, and consumption data are from the BEA. Comprehensive data

embed bilateral linkages through channels of consumption risk sharing in a multi-region theoretical framework. Compared to a symmetric two-economy model led by BKK, this multi-economy framework allows us to examine the aggregate influences across bilateral exchanges with different partners, with their substitutability and complementarity considered, on each economy's consumption. Compared to the quantitative spatial models surveyed by Redding and Rossi-Hansberg (2017), this RBC framework has the advantage of examining the second moments (variance and covariance) in addition to the first moment (level) of macroeconomic fundamentals, both of which are essential for welfare analysis.

Figure 1: Wyoming's Bilateral Ties with Other States



(a) Bilateral Trade (b) Bilateral Migration (c) Consumption Corr.

This figure plots the economic linkages between Wyoming (in white) and other states in the U.S. averaged over the sample period of 1997-2017. A darker color suggests a higher value of trade and migration flows (the sum of inflows and outflows) as well as a greater correlation coefficient of real consumption per capita. Data sources are listed in Appendix B.

We focus on state-level analysis within the US in this paper, but the general framework can be easily tailored to another setting of interest. In the empirical section, we explore the relationship between consumption risk sharing and geographic distance. Following the macroeconomic literature such as Asdrubali et al. (1996) and Kose et al. (2009), we measure a region's consumption risk sharing as the response of its relative consumption growth to its relative output growth. A greater response suggests a lower degree of consumption risk sharing, since the region's own income more predominantly drives its consumption fluctuations. We first calculate the degree of bilateral risk sharing using the output and consumption per capita data of the fifty US states over the period 1977-2019. In the next step we document that risk sharing is weaker for state pairs that are more geographically distant: Every 1% increase in distance lowers the response of relative consumption to output growth between a pair of states by 0.151 (or 0.402 standard deviations). This spatial characteristic of bilateral economic linkages echoes the

for state-to-state financial flows are not available to our knowledge.

prediction of the classic gravity model. As a novel empirical regularity of consumption, the finding points to the existence of barriers influenced by geography in the channels of risk sharing.

Furthermore, we examine the 2006 North Dakota oil boom as an event study to verify the importance of geography in determining the variation of consumption gains for other states. Through panel regressions, we find that due to the positive output shock, bilateral linkages of North Dakota with other states exhibit strong geographic patterns: North Dakota witnessed greater migration and trade inflows from states located in closer proximity. Meanwhile, these states also experienced stronger consumption comovements with North Dakota following the oil shock.

Motivated by the empirical findings, we develop a DSGE model to examine the channels that may shape this geographic pattern of consumption synchronization. Our model is populated by representative households who reside in different states. There are three forms of bilateral economic exchanges among states: trade, migration, and finance. In the trade channel, we follow the classic Armington (1969) model and assume each state produces one type of intermediate goods which are traded across states subject to iceberg trade costs. In the migration channel, we modify the framework developed by Artuc et al. (2010), who derive an Euler-type condition to capture dynamic labor adjustments. In our model, we assume households make forward-looking migration decisions in response to consumption differentials across states under migration frictions. Both the trade and migration models mentioned above have been adopted in the recent literature that examines the macroeconomic impacts of economic linkages (see, for example, Caliendo et al. (2018), House et al. (2018), and House et al. (2020)).

What is more unique about our spatial analysis is the modeling of financial flows in a multi-region framework. Due to the difficulty of incorporating a frictional financial channel in a multilateral model, existing literature has either focused on extreme scenarios (autarky or complete markets) or taken net asset positions directly from the data as exogenous. In contrast to these approaches, we set up a portfolio choice problem and endogenize households' preferences among assets from different states driven by their risk sharing needs. Furthermore, we introduce bilateral financial frictions as iceberg transaction costs on asset returns following Heathcote and Perri (2013) and Tille and Van Wincoop (2010).[3] To derive portfolios under frictions, we employ and extend

[3] An alternative modeling assumption of the financial friction is information asymmetry. Okawa and Van Wincoop (2012) discuss the comparability of information frictions and asset transaction costs in terms of their prediction for the gravity model of financial flows. Even within a country, there exist such financial frictions that covary with geography. A piece of empirical evidence for this is the 'home bias at

Devereux and Sutherland (2011)'s solution technique, which combines a second-order approximation of the Euler equation and a first-order approximation of other equations to derive the steady-state portfolio in a DSGE model. The portfolio choice will in turn affect consumption correlations, which allows us to quantify both the magnitude of bilateral financial frictions and the distortion of consumption caused by them.

To illustrate the mechanism of how the three channels interact with each other to jointly shape cross-state consumption correlations, we start with a symmetric two-economy framework à la BKK. The model features key elements of real business cycles including endogenous capital accumulation and labor supply. We enrich the framework by introducing multiple channels of risk sharing subject to frictions. By conducting a set of comparative analyses, we find that the interplay among the three channels of risk sharing may yield non-monotonic predictions of how the frictions in the channels affect consumption correlations across states. For example, higher financial frictions, by tilting portfolios towards domestic assets, reduce bilateral consumption correlations in general, consistent with the argument from the neoclassical model of risk sharing (Lucas (1982)). Nevertheless, when financial frictions are so high as to deteriorate wealth accumulations, population moves out of the region which has experienced a positive productivity shock. Meanwhile, the productivity shock leads to the region's terms-of-trade depreciation on impact which translates to lower wage rates. Therefore, these migration outflows which raise local wages due to decreased labor supply, will stabilize the cross-region wage disparity and lead to stronger consumption comovement. This analysis, by showing the effects of the channels' interactions on consumption, underscores the importance of examining these channels in an integrated general equilibrium setting.

After discussing the economic intuition of how the three channels affect macroeconomic dynamics using the two-economy model, we extend it to a multi-region framework for a quantitative assessment of the theory. In this numerical analysis, we still focus on the bilateral linkages built through the three channels between a pair of states. Meanwhile, we consider the rest of the economy (ROE) which exerts 'multilateral resistance' on the state-pair under examination in the spirit of Anderson and Van Wincoop (2003). Therefore, we develop a trilateral framework that consists of the state-pair and ROE which aggregates all the other states from the state-pair's perspective. This trilateral framework allows us to overcome the computational challenge of solving the portfolio choice problem in a DSGE model with many economies of uneven sizes. In terms of parametrization of the quantitative model, we calibrate trade and migration frictions

home' phenomenon documented by Coval and Moskowitz (1999).

to match a state-pair's bilateral trade and migration flows. Furthermore, we use the state-pair's coefficient of risk sharing estimated from the empirical section as a targeted moment to solve for the portfolio that supports this consumption comovement, and then recover the bilateral financial frictions from this specific portfolio arrangement. We conduct the estimation for all the state pairs in our sample, after which we confirm that the three types of bilateral frictions all show significantly positive correlations with bilateral geographic distance. For a 1% increase in distance, trade, migration, and financial frictions increase by 0.53%, 0.10%, and 0.23% respectively.

After computing the magnitude of frictions, we proceed to quantify their impacts on consumption. For this purpose, we conduct a series of counterfactual analyses where frictions are turned off. When evaluating the level of consumption in the steady state of the economy, we find that most states benefit from the reduction in trade costs, whereas the reduction in migration costs generates disparate predictions for different states. The most affluent states such as New York and California benefit from population inflows, while other states witness lower wage income under the labor market integration. In terms of second moments, eliminating three types of frictions uniformly leads to lower consumption volatility. The mean reduction in consumption volatility across states averaged across the state-pairs each state forms is 0.7%, 1.0%, and 0.3% respectively when bilateral trade, migration, and financial frictions are turned off. This result supports the argument that reducing barriers in the channels of risk sharing will yield welfare gains by smoothing consumption fluctuations. These counterfactual analyses not only disentangle the influences of each channel on the level and volatility of consumption, but also provide guidance for fiscal policies which, by mitigating the impacts of the frictions, reduce consumption inequality. Using an example that studies the direction and magnitude of transfers across states to alleviate the effects of trade costs on the level of consumption, we show that our framework can be a useful and flexible tool for the design of macroeconomic policies which aim to narrow consumption disparity across space and time.

This paper contributes to the macroeconomic literature on consumption risk sharing by adding the geographic dimension, which enriches the understanding of the patterns and determinants of consumption comovement across economies. To explain the failure of consumption risk sharing, existing international macroeconomic literature examines frictions in the financial market (e.g. Cole and Obstfeld (1991), Baxter and Crucini (1995), Kollmann (1995), and Lewis (1996)) and dynamics in the goods market (e.g. Dumas and Uppal (2001), Corsetti et al. (2008), and Eaton et al. (2016)) that impair consumption smoothing across countries. Nevertheless, many of these works focus on one channel only and therefore do not conduct a comprehensive analysis of multiple channels of risk sharing with their influences on consumption disentangled. Furthermore, most papers employ a two-country framework, which is not ideal to study the aggregate influences of bilateral linkages, with potential substitutability and complementarity, on macroeconomic fundamentals. There are two notable exceptions that are closer to our work. First, Fitzgerald (2012) disentangles the impacts of trade costs and financial frictions on cross-country risk sharing. Compared to her paper which captures countries' financial frictions as the departure of consumption allocation relative to a benchmark country (the US) from complete markets, our portfolio choice framework makes it possible to quantify the magnitude of financial frictions at the bilateral level easier for cross-sectional comparison and counterfactual analysis. Second, House et al. (2018) combine frictional trade and migration channels in a multi-region framework to quantify the benefits of labor mobility in the European Union. They have rich New Keynesian ingredients in the theoretical framework but they do not explicitly model financial frictions across economies, which are important in shaping the variation in bilateral consumption comovement in our risk-sharing analysis.

In the domestic context, Asdrubali et al. (1996), Hess and Shin (1998), Crucini (1999), Athanasoulis and Van Wincoop (2001), and Del Negro (2002) pioneered the work on consumption risk sharing within the US. These empirical works quantify the level of intranational risk sharing using state-level data. At the micro level, seminal papers including Storesletten et al. (2004) and Heathcote et al. (2014) explore heterogeneity across the US households in terms of the impacts of income on consumption. Neither these macro nor micro perspectives focus on the effects of bilateral economic linkages across regions or the influences of region-specific conditions on households' consumption and migration decisions. Therefore, our paper contributes to this literature by considering additional channels for facilitating consumption smoothing within a country.

This paper is also influenced by the recent development in the spatial economics literature. As is discussed in the comprehensive survey by Redding and Rossi-Hansberg (2017), new quantitative models of economic geography provide powerful yet tractable tools to characterize the distribution of economic activity across a large number of locations of uneven sizes. There are mainly two dimensions along which our work differs from and potentially contributes to that strand of literature. First, we add a financial channel under bilateral frictions by setting up the portfolio choice framework, which complements the existing papers that primarily focus on the real side of the economy consisting mainly of linkages in the goods and labor markets. Second, our RBC framework has the advantage of examining the second moments (variance and covariance) in addition to the first moment (level) of macroeconomic variables. For any risk-averse agent, both the level and the volatility of consumption are essential for welfare analysis. But the 'exact hat algebra' method widely used in the existing quantitative trade literature does not excel in analyzing the volatility of variables, especially when one 1) departs from the assumption of time-separable logarithm utility of consumption, 2) deviates from extreme cases for financial allocation including autarky or complete markets. Therefore, our framework fills the gap in the literature by endogenizing financial investment both over time and across space. Admittedly, the local solution method used in this RBC framework is not as flexible as the global method used in the quantitative trade literature, yet it proposes a new technique to incorporate a frictional financial channel in a multi-region framework.

Lastly, this paper contributes to the extensive empirical literature on the gravity model. Since being introduced by Isard (1954) and Tinbergen (1962), the model has emerged as a classic framework in the trade literature due to its success in matching bilateral trade flows. More recently, seminal works including Anderson and Van Wincoop (2003) and Eaton and Kortum (2002) refine the theoretical foundations of the framework that rationalize empirical regularities of bilateral trade. In addition to trade, the gravity model has been applied to a wide range of topics including financial assets (e.g. Portes and Rey (2005), Martin and Rey (2004), Aviat and Coeurdacier (2007), and Okawa and Van Wincoop (2012)) and population flows (e.g. Lewer and Van den Berg (2008) and Ramos and Suriñach (2017)). Nevertheless, less is known about the effects of distance on macroeconomic fundamentals. Our paper, together with Chertman et al. (2020) for cross-country analysis, adds to this literature by exploring the role of geographic distance in shaping the consumption pattern.

The remainder of the paper proceeds as follows: Section 2 empirically explores the influence of geographic distance on consumption comovement. Section 3 develops a two-economy framework to examine the three channels of consumption risk sharing influenced by distance. Section 4 conducts a quantitative assessment of a multi-region model to quantify the level and influence of frictions from these channels on consumption. Section 5 concludes.

2 Empirical Motivation

This section empirically establishes the importance of geographic distance for bilateral consumption risk sharing by using the US data. Our empirical analysis consists of two parts. First, we use the state-level consumption and output data to compute the degree of bilateral consumption risk sharing and find that it weakens with the geographic distance between a pair of states. Second, we examine the 2006 North Dakota oil shock as an event study to verify the role of geography in shaping the variation in consumption comovement of other states with North Dakota. The evidence points to the existence of frictions that covary with geography in the channels of risk sharing.

Following the literature including Asdrubali et al. (1996) and Kose et al. (2009), we measure a region's consumption risk sharing as the response of its relative consumption growth to its relative output growth. In particular, we focus on bilateral risk sharing so that we can exploit pair-specific factors including geographic distance in order to examine the patterns and determinants of consumption comvement across regions. Specifically, we evaluate risk sharing between state i and j from

$$\Delta \log\ c_{it} - \Delta \log\ c_{jt} = \alpha_{ij} + \beta_{ij}(\Delta \log\ y_{it} - \Delta \log\ y_{jt}) + \epsilon_{ijt}, \tag{1}$$

where $\Delta \log\ c_{it}$ ($\Delta \log\ c_{jt}$) denotes the growth of log real per-capita consumption of state $i(j)$ at time t, and $\Delta \log\ y_{it}$ ($\Delta \log\ y_{jt}$) denotes the growth of log real per-capita output. The coefficient β_{ij} measures the degree of bilateral consumption risk sharing. In the case with perfect risk sharing, relative consumption growth should equal zero regardless of relative output growth, which yields a coefficient of 0. In the opposite case with complete autarky, a state's consumption is solely determined by its own output, which implies a coefficient of 1. Therefore, a lower value for the coefficient β_{ij} suggests a higher degree of bilateral risk sharing.

The data using which we evaluate equation 1 are obtained from the following sources. The US Bureau of Economic Analysis (BEA) reports state-level output, consumption and price data in the Regional Economic Accounts (REA). Our sample spans from 1977 to 2019 during which period the data for real state gross state product (GSP) are available. State-level consumption and price data from the BEA have shorter coverage (from 1997 and 2008 onwards respectively), which are not ideal for our analysis of risk sharing that requires long-horizon data. Therefore, we follow Asdrubali et al. (1996)'s method of constructing state-level consumption by rescaling state-level retail sales by the country-level ratio of private consumption to retail sales, both of which are available from the BEA. Moreover, we obtain the state-level inflation series from Nakamura and Steinsson (2014) for the period of 1966-2008, deflated by which we obtain state-level real consumption. Appendix B provides the details of these datasets and describes the method of how we compile and analyze the data.

Table 1 provides a first glance at the state-level data of interest. Panel A reports the summary statistics of real output and consumption per capita of the 50 US states averaged from 1977-2019. The mean value of real output per capita across states is \$41,701 with a standard deviation of \$8,409. The median value is \$40,129, representing the mean output of Ohio and Georgia. In terms of consumption, the mean value across states is \$28,944 and the standard deviation is \$2,481. Both values are significantly lower for consumption than for output. The median states are Alaska and California, whose average consumption is \$28,815.

Panel B of table 1 presents bilateral correlations among all the state pairs over the sample period. The correlations are calculated using HP-filtered consumption and output per capita both in the logarithmic form. From the table, the mean bilateral output correlation is 0.422 and the consumption correlation is 0.340. This finding that bilateral output correlation is higher than consumption correlation across states within the US is consistent with international evidence documented by Lewis (1996), Backus et al. (1992), and Heathcote and Perri (2004) among others. Since this empirical regularity contradicts the theoretical prediction in complete markets, it remains to be a perplexing puzzle in international macroeconomics (Obstfeld and Rogoff (2000)). In this paper we use domestic data to quantify the degree of risk sharing and explore its determinants, which will potentially shed light on this consumption correlation puzzle in the international context as well.

We establish an empirical gravity model of consumption risk sharing by deriving a cross-sectional prediction for consumption comovement across states. In particular, we explore the implications of geographic distance for bilateral consumption risk sharing by conducting a two-stage regression. In the first stage, we follow equation 1 to estimate the bilateral risk-sharing coefficients for all the state pairs over the sample period. Table 2 summarizes the statistics of the estimated coefficients $\hat{\beta}_{ij}$. The mean and median values are 0.515 and 0.501 respectively. The fact that $\hat{\beta}_{ij}$ is between 0 and 1 implies imperfect consumption risk sharing across states.

In the second stage, we regress the estimated $\hat{\beta}_{ij}$ on the log of bilateral geographic distance:

$$\hat{\beta}_{ij} = \alpha + \gamma \log(dist_{ij}) + \Gamma X_{ij} + \nu_{ij}. \tag{2}$$

We also include gravity controls (X_{ij}) including state-pairs' products of time-averaged GSP, population, and GSP volatility in the regression. Our hypothesis is that state pairs with greater geographic distance exhibit weaker consumption risk sharing, since bilateral economic exchanges which facilitate consumption comovements potentially face frictions that increase with bilateral distance. γ in equation 2 is therefore expected to be positive under this hypothesis. When constructing the cross-state geographic distance, we apply the Haversine formula to state capitals' longitude and latitude to approximate the distance between two states. In addition, we use the shipment distance from the Commodity Flow Survey (CFS) and confirm the robustness of our empirical findings (shown in table A.3).[4]

Table 1: Summary Statistics of Real Output and Consumption

	Mean	Std. Dev.	Min	Median	Max	Obs.
A. Level (in Dollars)						
Output	41,701	8,409	28,311	40,129	73,551	50
Consumption	28,944	2,481	24,480	28,815	34,805	50
B. Bilateral Correlation						
Output	.422	.316	-.552	.947	.479	1,225
Consumption	.340	.329	-.511	.949	.388	1,225

Real output and consumption per capita are averaged over 1977-2019 for each state. Bilateral correlation of output (consumption) is calculated as the correlation of HP-filtered real output (consumption) per capita in logarithms among all the state pairs over the sample period.

The results reported in table 3 confirm our hypothesis that bilateral geographic distance and risk-sharing coefficients are significantly and positively correlated. In column (1), when distance rises by 1%, bilateral risk sharing weakens by 0.151 (or 0.402 standard deviations). In column (2) we exclude two non-contiguous states Alaska and Hawaii to reduce the potential bias caused by their peculiar geographic locations. We find that the coefficient for distance remains significantly positive. In column (3) we control for state-pairs' GSP per capita averaged over the sample period and find that risk sharing

Table 2: Summary Statistics of the estimated risk sharing coefficients

	Mean	Std. Dev.	Median	Obs.
$\hat{\beta}_{ij}$	0.515	0.292	0.501	1,225

β_{ij} is estimated as the response of the relative consumption growth to the relative output growth as specified in equation 1. A higher β_{ij} suggests a lower degree of risk sharing.

[4]The CFS reports the shipment mileage between origin and destination ZIP code points for commodity flows used for domestic expenditure within the US. We use the average mileage of shipments between two states to calculate this CFS-based bilateral distance.

Table 3: Gravity Model of Risk Sharing

Dep. Var: $\hat{\beta}_{ij}$	(1)	(2)	(3)	(4)
$\log(d_{ij})$	0.151 ***	0.162 ***	0.156 ***	0.166 ***
	(0.010)	(0.010)	(0.010)	(0.010)
$\log(\bar{y}_1 \cdot \bar{y}_2)$			-0.099 ***	-0.121 ***
			(0.032)	(0.037)
$\log(\sigma(y_1) \cdot \sigma(y_2))$				0.019
				(0.023)
$\log(\bar{N}_1 \cdot \bar{N}_2)$				0.029 ***
				(0.005)
Sample	all states	mainland	all states	all states
Obs.	1225	1128	1225	1225
R^2	0.161	0.171	0.169	0.192

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the risk sharing coefficient $\hat{\beta}_{ij}$, which is estimated using the real consumption and output data over 1977 – 2019. $\log d_{ij}$ denotes the geographic distance between state i and j in logarithms. y_i and N_i denote real output per capita and population in logarithms of state i. $\bar{x}$ and $\sigma(x)$ represent the mean and volatility of a variable x over the sample period.

is stronger for states with higher income levels. Therefore, bilateral risk sharing covaries with distance and income per capita in the same direction as trade flows in the classic gravity model. In column (4) we add other controls including the product of GSP volatility, and that of population averaged over time. The coefficient estimates imply that it is easier to achieve risk sharing for state-pairs with a smaller population, while output volatility does not show significant relevance. Meanwhile, the signs of the coefficients for distance and output per-capita remain the same as in column (3).

In addition the baseline estimation, we perform a series of tests to verify the robustness of our finding. In particular, we consider alternative data sources for state-level consumption and inflation, as well as for bilateral geographic distance. Moreover, we reconstruct the measure of bilateral risk sharing after controlling for states' distinct exposure to aggregate risks. The results reported in Appendix A suggest that our finding about the gravity model of consumption risk sharing remains robust.

After exploring the general covariance between bilateral risk sharing and geographic distance using long-term data, we conduct an event study to verify the importance of geography for bilateral economic linkages including consumption comovement. Specifically, we focus on the North Dakota oil supply shock that started from the surprising discovery of oil by a petroleum geologist in 2006. The discovery provides a natural experiment for us to evaluate the impacts of a local output boost. The rapid oil extraction since the discovery has not only fueled the economic boom of North Dakota (ND hereafter) but also positively affected other states through their economic ties with ND.

To establish the spatial feature of economic linkages in the wake of the oil shock, we run a panel regression with all the state pairs formed by ND over the period from 1991 to 2019 where migration and trade data are available. The regression is specified as follows

$$X_{ijt} = \alpha_0 + \alpha_1 \mathrm{Oil}_t + \sum_{m=1}^{T} \alpha_{2m} \mathrm{Oil}_{t-m} + \alpha_3 \log(dist_{ij}) + \sum_{n=0}^{T} \alpha_{4n} \mathrm{Oil}_{t-n} \times \log(dist_{ij}) + \alpha_{5t} I_t + \epsilon_{ijt}. \tag{3}$$

X_{ijt} represents bilateral variables of interest including migration flows ($\log(mig_{ijt})$), trade values ($\log(trd_{ijt})$), and relative per-capita consumption growth between state i as ND and j as any other state. For migration and trade, we focus on ND's population and goods inflows from other states to capture the spillover of the positive shock. For the consumption growth, we consider both $\Delta c_{ijt} \equiv \Delta \log c_{it} - \Delta \log c_{jt}$ and $\Delta \tilde{c}_{ijt} \equiv (\Delta \log c_{it} - \Delta \log c_{jt}) - (\Delta \log y_{it} - \Delta \log y_{jt})$. The latter can be regarded as the consumption growth unexplained by the output growth of ND relative to other states, which provides a more robust measure of consumption risk sharing. To isolate the responses of these variables to the oil shock as deviations from their long-term trend, we take the difference between the realization of these bilateral variables at time t and their mean values over the sample period, and use these demeaned values for the dependent variables. Among the independent variables, we control for time fixed effects (denoted as I_t) which reflect the aggregate shocks that happen at time t. Furthermore, Oil_t is a binary variable which is unity when t denotes year 2006 and zero otherwise. We also consider medium-run effects of the shock by including lagged dummies Oil_{t-m} which equal one when the oil shock happens m years ago. In the baseline case, we set the maximum number of lags as three years for migration and consumption, and as eleven years for trade to get sufficient observations under its five-year data frequency. The key variable of interest to verify the importance of geography for economic linkages is $\sum_{n=0}^{T} \alpha_{4n}$, the linear combination of coefficient estimates for the interaction terms of the oil shock and bilateral distance.

Table 4 reports the regression results. Based on the coefficient estimates for the interaction terms, bilateral economic linkages exhibit strong spatial patterns. As is shown in columns (1) and (2), a 1% increase in bilateral geographic distance lowers migration and trade flows from another state to ND by 0.394% and 0.578% respectively due to the oil

Table 4: Bilateral Linkages after the Oil Shock

Dep. Var:	(1) log(mig)	(2) log(trd)	(3) Δc	(4) $\Delta\tilde{c}$
Oil_t	0.124		-0.009	0.014
	(0.465)		(0.049)	(0.054)
$\sum_{m=1}^{T} Oil_{t-m}$	-0.974	1.883 *	-0.045	0.098
	(0.599)	(0.967)	(0.077)	(0.063)
log($dist$)	0.013	0.012	-0.002	-0.001
	(0.014)	(0.075)	(0.002)	(0.002)
$\sum_{n=0}^{T} Oil_{t-n} \times \log(dist)$	-0.394 ***	-0.578 *	0.049 ***	0.040 **
	(0.146)	(0.325)	(0.017)	(0.017)
Observations	1,360	244	1,372	1,372
R^2	0.645	0.657	0.650	0.676

Robust standard errors in parentheses. *** significant at 1%, ** at 5%, and * at 10%. The dependent variables include North Dakota (ND)'s migration (log(mig)) and trade (log(trd)) inflows from other states, as well as ND's consumption growth relative to other states (Δc), and the relative consumption adjusted for output growth ($\Delta\tilde{c}$). log($dist$) denotes the geographic distance between ND and other states. Oil_t is a dummy variable for the oil shock to ND in 2006. Its coefficient is missing in column (2) since trade data from the CFS are not available that year.

shock. This finding points to the barriers in these two channels that covary with geography which limit the scope of positive influences brought forth by ND's economic success. Consequently, residents from distant states are constrained from physically moving to or selling products to the booming state. Such barriers can also account for the spatial pattern of consumption. As is reported in columns (3) and (4), ND's per-capita consumption growth is larger in magnitude relative to that of geographically distant states. From column (3), a 1% increase in distance raises ND's relative consumption boost driven by its oil shock by 0.049%. This result, which suggests that ND's consumption is more synchronized with neighboring states', indicates that geography plays an essential role in shaping the variation in consumption comovement. The result remains robust in column (4) where we adjust consumption for output differentials, which further implies that the degree of consumption risk sharing decreases in distance across economies, consistent with the empirical regularity we documented earlier.

To conclude the empirical section, we first use consumption and output data to compute the degree of bilateral consumption risk sharing over a long horizon across states in the US. Furthermore, we establish a gravity model by documenting that risk-sharing deteriorates as geographic distance rises between a pair of states. In addition to this general pattern, we conduct an event study examining the 2006 North Dakota oil shock to verify that distance plays an essential role in spreading consumption gains, potentially through channels including migration and trade. These findings point to the existence of frictions in the channels of risk sharing that covary with distance. In the next section, we develop a theoretical model in which we examine the interplay among the channels and quantify their impacts on the level and comovement of consumption across economies.

3 Theoretical Model

3.1 Setup

In this section we develop a theoretical framework to examine the channels of consumption risk sharing across regions. The economy is populated by a continuum of infinitely-lived homogeneous households who reside in different regions indexed $i \in [1, \mathcal{I}]$. Regions are interconnected through trade, migration, and finance channels.

Each region produces two intermediate goods: tradables (T) and nontradables (NT). The production of intermediate goods in sector $s \in \{T, NT\}$ combines capital $K_{is,t}$ and labor $L_{is,t}$ with a Cobb-Douglas technology:

$$Y_{is,t} = A_{i,t} K_{is,t}^{\alpha} L_{is,t}^{1-\alpha}. \tag{4}$$

The region-specific productivity $A_{i,t}$ is subject to shocks $\epsilon_{i,t}$. To capture the comovement of productivity shocks across regions, we specify a joint autoregression for the vector of productivity $A_t \equiv (A_{1,t}, A_{2,t}, ..., A_{\mathcal{I},t})$ subject to shocks $\epsilon_t \equiv (\epsilon_{1,t}, \epsilon_{2,t}, ..., \epsilon_{\mathcal{I},t})$:

$$A_t = \rho A_{t-1} + \epsilon_t, \tag{5}$$

where ρ is a persistence coefficient matrix for lagged productivity of all the regions. The contemporaneous correlations among regional shocks $\epsilon_{i,t}$ can be captured by a covariance matrix denoted as Σ.

The final goods for consumption consist of tradables $C_{iT,t}$ and nontradables $C_{iNT,t}$:

$$C_{i,t} = C_{iT,t}^{\nu} C_{iNT,t}^{1-\nu}, \tag{6}$$

where ν captures the weight of tradables which are composed of intermediate goods supplied by all the regions. The final goods for investment, whose price and quantity are denoted as $I_{i,t}$ and $P_{Ii,t}$, are also a Cobb-Douglas composite:

$$I_{i,t} = I_{iT,t}^{\nu_I} I_{iNT,t}^{1-\nu_I}, \tag{7}$$

where investment adds to the capital stock in region i net of depreciation δ

$$K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t}. \tag{8}$$

The market clearing conditions for factors of production and for nontradable goods in region i are respectively given by

$$K_{i,t} = K_{iT,t} + K_{iNT,t}, \qquad L_{i,t} = L_{iT,t} + L_{iNT,t}, \tag{9}$$

$$Y_{iNT,t} = C_{iNT,t} + I_{iNT,t}. \tag{10}$$

On the other hand, tradable goods for consumption and investment in region i will be a CES bundle of intermediate tradable goods sourced from all the regions:

$$X_{iT,t} = C_{iT,t} + I_{iT,t}, \qquad \text{where} \qquad X_{iT,t} = (\sum_{j=1}^{\mathcal{I}} X_{ji,t}^{\frac{\phi-1}{\phi}})^{\frac{\phi}{\phi-1}}. \tag{11}$$

However, trade from j to i is subject to an iceberg cost $\tau_{ji} \geq 1$. Therefore, the aggregate price level of tradables in region i is determined by the trade cost, as well as the price of j's output, denoted as $p_{j,t}$ summed across regions of origin:

$$P_{iT,t} = [\sum_{j=1}^{\mathcal{I}} (\tau_{ji} p_{j,t})^{1-\phi}]^{\frac{1}{1-\phi}}. \tag{12}$$

Bilateral trade flows from j to i at t will therefore be given by

$$X_{ji,t} = \pi_{ji,t} X_{iT,t}, \qquad where \qquad \pi_{ji,t} = (\frac{\tau_{ji} p_{j,t}}{P_{iT,t}})^{-\phi}. \tag{13}$$

In addition, trade costs also enter the tradable goods' market clearing condition

$$Y_{iT,t} = \sum_{j}^{\mathcal{I}} \tau_{ij} X_{ij,t}. \tag{14}$$

In addition to trade, regions are integrated in the finance channel by holding each other's assets, whose dividend payout is calculated as capital income net of investment expenditure. Let $p_{i,t}$ be the price and $Y_{i,t} = Y_{iT,t} + Y_{iNT,t}$ be the quantity of output in region i, and α be the capital share from the Cobb-Douglas production function, region i's dividend at time t is given by

$$D_{i,t} = \alpha p_{i,t} Y_{i,t} - P_{Ii,t} I_{i,t}. \tag{15}$$

The returns to the assets from region i include these dividends and the changes in asset prices denoted as $q_{i,t}$:

$$R_{i,s,t} = \frac{q_{i,s,t} + D_{i,s,t}}{q_{i,s,t-1}}. \tag{16}$$

In every region there is a mutual fund that constructs a portfolio of assets from different regions on behalf of the households in that region. A household has the right to an equal share of the fund as long it resides there. To simplify the portfolio choice problem, we assume households are myopic and do not take their migration probabilities into consideration. Instead, they expect themselves to stay and consume in the region of residence when deciding on investment for the next period.[5] Meanwhile, they incur costs when collecting financial gains earned from other regions. In particular, the literature on the gravity model of financial flows across countries, led by Portes and Rey (2005) and Okawa and Van Wincoop (2012), suggests that bilateral financial frictions covary with geographic distance. In this spirit, we introduce bilateral financial friction $e^{-f_{ij}}$ as an iceberg trade cost region j incurs when repatriating financial gains from region i. The cost can be regarded as an asset transaction cost or tax, similar to the friction modeled in Heathcote and Perri (2004) and Tille and Van Wincoop (2010).[6] Moreover, we assume it is second order in magnitude (i.e. proportional to shocks in the model). This assumption allows us to use the perturbation method developed by Devereux and Sutherland (2011) to solve the portfolio choice problem. The method combines a second-order approximation of the Euler equations and a first-order approximation of other equations in the model. Specifically, region i's Euler equation follows

$$E_t[\frac{U'(c_{i,t+1})}{P_{i,t+1}} R_{i,t+1}] = E_t[\frac{U'(c_{i,t+1})}{P_{i,t+1}} e^{-f_{ji}} R_{j,t+1}], \qquad \forall j \in [1, \mathcal{I}]. \tag{17}$$

[5] A future extension of this baseline scenario is to relax the assumption and allow households to consider migration probabilities which prompt them to reduce saving and raise current consumption when making the investment decisions.

[6] Okawa and Van Wincoop (2012) discuss alternative bilateral financial frictions, including information costs, which can also rationalize the gravity model of financial flows.

where $c_{i,t}$ denotes consumption per-capita.

We use the Euler equation to derive the solution to the portfolio choice problem. First, we assume assets from region $\mathcal{I}$ to be a numeraire asset and denote i's holding of j's assets as $\alpha_{j,i,t}$. Region i's external wealth position is therefore given by

$$\mathcal{W}_{i,t+1} = R_{\mathcal{I},t}\mathcal{W}_{i,t} + \sum_{j}^{\mathcal{I}} \alpha_{j,i,t}(e^{-f_{ji}}R_{j,t} - e^{-f_{\mathcal{I}i}}R_{\mathcal{I},t}) + p_{i,t}\sum_{s} Y_{is,t} + T_{i,t} - P_{i,t}C_{i,t} - P_{Ii,t}I_{i,t}, \tag{18}$$

where $T_{i,t}$ denotes the tax transfer region i receives, which is introduced to capture fiscal policies that potentially also play an essential role in risk sharing within a country.

The vector of excess returns to the other assets is introduced as R_x:

$$\hat{R}'_{x,t} = [\hat{R}_{1,t} - \hat{R}_{\mathcal{I},t}, \hat{R}_{2,t} - \hat{R}_{\mathcal{I},t}, ..., \hat{R}_{\mathcal{I}-1,t} - \hat{R}_{\mathcal{I},t}], \tag{19}$$

where $\hat{y}_t$ represents the log-deviation of any variable y from its steady state at t. Next, we evaluate the second-order Taylor expansion of the Euler equation 17 as

$$E_t[\hat{R}_{x,t+1} + \frac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{i,t+1} + \hat{P}_{i,t+1})\hat{R}_{x,t+1}] = -\frac{1}{2}\mathcal{F}_i + \mathcal{O}(\epsilon^3), \tag{20}$$

where $\hat{R}^{2'}_{x,t+1}$ denotes the vector of excess squared returns

$$\hat{R}^{2'}_{x,t+1} = [\hat{R}^2_{1,t+1} - \hat{R}^2_{\mathcal{I},t+1}, \hat{R}^2_{2,t+1} - \hat{R}^2_{\mathcal{I},t+1}, ..., \hat{R}^2_{\mathcal{I}-1,t+1} - \hat{R}^2_{\mathcal{I},t+1}], \tag{21}$$

and $\mathcal{F}_i$ denotes i's vector of financial frictions defined as

$$\mathcal{F}'_i = [f_{\mathcal{I}i} - f_{1i}, f_{\mathcal{I}i} - f_{2i}, ..., f_{\mathcal{I}i} - f_{\mathcal{I}-1i}], \tag{22}$$

whose k^{th} element represents the additional financial friction region i incurs when holding $\mathcal{I}$'s asset relative to k's. The last term in equation 20, $\mathcal{O}(\epsilon^3)$, captures all terms of order higher than two.

In the next step, we take the difference of any pair of regions i and j's expanded Euler equations (20)

$$E_t[\sigma(\hat{c}_{i,t+1} - \hat{c}_{j,t+1}) + (\hat{P}_{i,t+1} - \hat{P}_{j,t+1})]\hat{R}_{x,t+1} = \frac{1}{2}(\mathcal{F}_i - \mathcal{F}_j). \tag{23}$$

The term in the bracket represents the inflation-adjusted consumption differential across regions. We denote it in the vector term for all the region-pairs under examination as $\hat{cp}$ hereafter. Equation 23 can therefore be written as

$$E_t(\hat{cp}_{t+1}\hat{R}'_{x,t+1}) = \frac{1}{2}\mathcal{F} + \mathcal{O}(\epsilon^3), \tag{24}$$

where $\mathcal{F}$ stacks $\mathcal{F}'_i - \mathcal{F}'_j$ vertically in a $\frac{\mathcal{I}(\mathcal{I}-1)}{2} \times (\mathcal{I} - 1)$ matrix for the $\frac{\mathcal{I}(\mathcal{I}-1)}{2}$ region-pairs being analyzed. Appendix C outlines the technical details of how we solve the portfolio choice problem by evaluating equation 24, the portfolio determination condition. From this equation, one can tell that bilateral financial frictions in $\mathcal{F}$ affect cross-region consumption comovement $\hat{cp}$ through asset allocations.

Households' objective is to maximize their expected lifetime utility. At the beginning of every period, a household living in region i supplies labor, collects wage and financial income, and decides on consumption and investment. It derives utility from consumption $c_{i,t} = \frac{C_{i,t}}{N_{i,t}}$ and disutility from labor hours $l_{i,t} = \frac{L_{i,t}}{N_{i,t}}$ in its region of residence:

$$U_{i,t} = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \kappa\frac{l_{i,t}^{1+\eta}}{1+\eta}, \tag{25}$$

where σ captures the degree of risk aversion and $\frac{1}{\eta}$ is the elasticity of labor supply.

After earning and spending its income in region i, the household decides whether and where it wants to migrate. When it makes the decision at time t, it takes into account a non-pecuniary migration cost $d_{ij} > 0$ when moving from region i to j. If it stays, the cost is normalized to zero ($d_{ii} = 0$). Whether the household stays or moves, it collects an idiosyncratic benefit $\omega_i \sim F(\omega)$ from being located in region i at the end of the period. ω_i can be considered as a non-monetary benefit, such as weather and culture, that adds to the utility of living in region i. Following Artuc et al. (2010), we assume ω_i is i.i.d across households, time, and space. It is drawn from an extreme-value distribution with zero mean:

$$F(\omega) = \exp[-e^{\omega/\theta-\gamma}]. \tag{26}$$

Therefore, a household's expected value of being in region i at time t is

$$V_{i,t} = U_{i,t} + \beta E(V_{i,t+1}) + \sum_{j}^{\mathcal{I}} \int (\bar{\omega}_{ij,t} + \omega_{jt}) f(\omega_j) \Pi_{k\neq j} F(\bar{\omega}_{ij,t} - \bar{\omega}_{ik,t} + \omega_{jt}) d\omega_j. \tag{27}$$

From the three components on the right side of the equation, the expected value consists of the current utility the household obtains, the base value of staying in the region, and option value of moving from the region to others in the future. $\bar{\omega}_{ij,t}$ denotes the cutoff benefit that makes the household indifferent between staying in i and moving to j at t:

$$\bar{\omega}_{ij,t} \equiv \beta[E(V_{j,t+1}) - E(V_{i,t+1})] - d_{ij,t}. \tag{28}$$

Under the distributional assumption of ω, the share of population moving from i to j at t follows

$$m_{ij,t} = \frac{\exp(\bar{\omega}_{ij,t}/\theta)}{\sum_{k=1}^{\mathcal{I}} \exp(\bar{\omega}_{ik,t}/\theta)}, \tag{29}$$

where the parameter from the extreme-value distribution θ governs the responsiveness of migration to economic conditions. The law of motion for population in region i given $m_{ij,t}$ follows

$$N_{i,t} = \sum_{j=1}^{\mathcal{I}} m_{ji,t-1} N_{j,t-1}. \tag{30}$$

With all the ingredients introduced, we now proceed to characterize optimal consumption risk sharing across regions as a benchmark. Suppose there is a benevolent social planner whose objective is to maximize the sum of all the representative households' expected lifetime utility in the economy:

$$\max \sum_{t=0}^{\infty} \sum_{i}^{\mathcal{I}} \beta^t N_{i,t} \lambda_{i,t} \left(\frac{c_{i,t}^{1-\sigma}}{1-\sigma} - \kappa \frac{l_{i,t}^{1+\eta}}{1-\eta}\right) \tag{31}$$

subject to the resource constraint

$$\sum_{i}^{\mathcal{I}} (N_{i,t} P_{i,t} c_{i,t} + P_{Ii,t} I_{i,t}) = \sum_{i}^{\mathcal{I}} p_{i,t}(Y_{iN,t} + \sum_{j}^{\mathcal{I}} X_{ij,t}) + \sum_{i}^{\mathcal{I}} T_{i,t}. \tag{32}$$

$\lambda_{i,t}$ is the per capita weight that the social planner assigns to the utility of residents in region i at time t. The social planner's optimal decision rule for a pair of regions i and j should satisfy

$$\frac{\lambda_{i,t}}{\lambda_{j,t}} \left(\frac{c_{i,t}}{c_{j,t}}\right)^{-\sigma} = \frac{P_{i,t}}{P_{j,t}}. \tag{33}$$

When asset markets are complete, the optimal consumption allocation in the competitive equilibrium coincides with the decision of the planner who assigns time-invariant weights to all the regions regardless of the realization of regional productivity shocks. Therefore, the ratio of $\lambda_{i,t}$ to $\lambda_{j,t}$ denoted as

$$\Lambda_{ij,t} = \frac{\lambda_{i,t}}{\lambda_{j,t}} \tag{34}$$

should be constant. Based on this analysis, the volatility of $\Lambda_{ij,t}$ over time reflects bilateral financial frictions because it captures the departure of consumption from the allocation under complete markets. As is argued by Fitzgerald (2012), $\Lambda_{ij,t}$ offers great flexibility since it does not depend on the assumption about the asset market structure or about the specific form the financial friction takes. However, it is easier to use the asset transaction cost f_{ij} we introduced earlier as a measure to quantify the magnitude of financial frictions for cross-region comparison and counterfactual exercise. Therefore, we will use $\Lambda_{ij,t}$ in the qualitative analysis and f_{ij} in the quantitative analysis in the next section for the examination of a two-region scenario.

3.2 Two-region Analysis

After describing the general setup including $\mathcal{I}$ regions, we analyze a two-region case to explain the mechanism through which different channels affect consumption risk sharing and illustrate how the channels interact with each other.

Before showing the quantitative results from numerical exercises, we conduct qualitative analysis to elucidate the intuition of how consumption risk sharing is achieved in a two-region framework. To keep this qualitative analysis tractable, we impose several simplifying assumptions temporarily: The two regions under examination, indexed 1 and 2, are perfectly symmetric. There is no endogenous labor supply, tax transfer, or capital accumulation. All goods are tradable subject to bilateral trade costs $\tau_{12} = \tau_{21} = \tau > 1$. Under these assumptions we analyze the cross-region ratio of any variable $x \equiv \frac{x_1}{x_2}$ whose deviation from the steady state is denoted as $\hat{x} = \log \frac{x - \bar{x}}{\bar{x}}$. Log-linearizing the goods market clearing condition (equations 13 and 14) and the social planner's allocation rule (equations 33 and 34) yields

$$\begin{aligned} \hat{Y} &= \Omega(1 - \sigma\phi)\hat{c} - \phi\hat{p} + \Omega\phi\hat{\lambda} + \Omega\hat{L}, \\ \textit{where} \quad \Omega &= \frac{1 - \tau^{1-\phi}}{1 + \tau^{1-\phi}}. \end{aligned} \tag{35}$$

Based on equation 35, the response of relative per-capita consumption $\hat{c}$ to relative output $\hat{Y}$ driven by productivity changes, varies with trade costs τ through the coefficient Ω. When domestic and foreign goods are sufficiently substitutable ($\phi > 1$), higher trade costs impede consumption risk sharing because the relative consumption increases with relative output fluctuations:

$$\frac{\partial(\partial c/\partial Y)}{\partial \tau} = \frac{1}{\sigma\phi - 1}\frac{1}{\Omega^2}\frac{\partial \Omega}{\partial \tau} > 0. \tag{36}$$

Meanwhile, three channels, represented by the other terms on the right hand side of 35, help absorb the impact of productivity shocks on consumption. In particular, the direction for the dynamics of the variables follows

$$\frac{\partial p}{\partial Y} < 0, \qquad \frac{\partial \lambda}{\partial Y} > 0, \qquad \frac{\partial L}{\partial Y} > 0. \tag{37}$$

To explain the economic interpretation of how these channels counteract output shocks to insulate consumption, we analyze a scenario where there is a relative negative output shock to region 1 ($\hat{Y} \downarrow$). First, a terms-of-trade appreciation ($\hat{p} \uparrow$) alleviates the shortfall of region 1's income and hence leaves its consumption less affected. Second, more financial resources, represented by $\hat{\lambda} \downarrow$, mitigates region 1's consumption fluctuation. Since λ can be interpreted as the inverse of marginal utility from consumption given price levels, the decline of λ's value represents the situation where the marginal utility of the residents in region 1 which inflicts the output loss is more valued by the social planner when allocating financial resources. Given this financial allocation, region 1's relative consumption does not decline as significantly. Third, migration of population out of region 1 ($\hat{L} \downarrow$) reduces the local population among which resources are allocated and therefore equalizes consumption per-capita across regions.

We now proceed to conduct numerical exercises and analyze the quantitative results of the model. The framework is similar in style to the workhorse model in international macroeconomics developed by Backus et al. (1992) who examine the real business cycles of two symmetric economies. We enrich the framework by incorporating trade, migration, and asset flows under frictions across economies. In terms of parameterization, the model is calibrated to the U.S. annual data for cross-state analysis. Table 5 summarizes the parametric assumptions under which the baseline two-region framework is solved. First, we adopt the standard assumptions from macroeconomic literature (listed in panel (I)) including the coefficient of risk aversion and elasticity of labor supply.

In panel (II), we report the parameters estimated from the U.S. aggregate economy. Specifically, we estimate labor share in production $1-\alpha$ to be 0.59 by dividing the labor earnings by the output data, both from the BEA, over the period of 1977-2019. In addition, we set the share of consumption expenditure on tradables (ν) as 0.31 following Johnson (2017), who estimates the value based on the US CPI expenditure data from the BEA. Moreover, we set the weight of tradables in investment (ν_I) as 0.4 following Bems (2008). His analysis uses the input-output table from the OECD. Last but not least, we follow Simonovska and Waugh (2014) and Artuc et al. (2010) when setting elasticities of trade and migration as 4.1 and 4.5 respectively.

In panel (III), we characterize the joint productivity process of a pair of states. We choose Georgia and Ohio (GA and OH for brevity), the median states in terms of output per capita, as our sample of analysis. We first calculate the total factor productivity (TFP) proxied by the Solow residual in each region $i \in \{\text{GA, OH}\}$ at time t from

$$\log(A_{i,t}) = \log(Y_{i,t}) - \alpha \log(K_{i,t}) - (1-\alpha)\log(L_{i,t}), \tag{38}$$

where $Y_{i,t}$ and $L_{i,t}$ are output and number of employees in state i in year t from the BEA over the sample period. State-level capital stock $K_{i,t}$ is not directly available, so we construct the measure following Garofalo and Yamarik (2002)'s method. Specifically, we apportion national capital stock to states based on their industry-level income data (see Appendix B for details). After we calculate the state-level TFP, we detrend the series with the Hodrick-Prescott (HP) filter. Lastly, we estimate a joint AR(1) process (specified in equation 5) assuming the shocks are serially independent and drawn from a joint normal distribution. Table 5 reports the persistence and covariance matrices of Georgia and Ohio's productivity.

Panel (IV) of table 5 lists the values of bilateral frictions calibrated to the pair of states under examination. Trade costs, migration costs, and financial costs are estimated to match three targeted moments: the mean export-to-output ratio (0.392), the mean emigrant-to-population ratio (0.028), and the bilateral consumption correlation (0.824) of Georgia and Ohio over the sample period. When estimating trade and migration frictions simultaneously, we start with an initial guess for the combination of the two frictions, solve for the corresponding wage rates and labor hours given the frictions that satisfy the labor market clearing condition (equation 9), and update the guess as well as repeat the procedure until the model-predicted export-to-output and emigrant-to-population ratios converge to those in the data. After calibrating these two frictions on the real side of the economy, we estimate model-consistent financial frictions. Unlike trade and migration whose bilateral flows are available in the data, state-to-state financial flows are not directly observable. Therefore, we infer financial frictions indirectly from the consumption pattern. Besides its feasibility given limited data availability, this method is helpful in capturing the influences of the financial channel on consumption comovement. Calibrating financial frictions with this method involves three steps. First, we obtain the coefficient matrices necessary to solve the portfolio choice problem from the first-order conditions of the model.[7] Second, we solve for asset holdings $\tilde{\alpha}$ under which the model-implied bilateral consumption correlation exactly matches that in the data. Third, we plug the calibrated portfolio in equation 24 to recover financial frictions.

Table 5: Parametrization

Parameter	Description	Value	Source
	(I)		
β	Annual discount factor	0.95	
σ	Coefficient of relative risk aversion	1	Macroeconomic
δ	Capital depreciation	0.06	Literature
η	Inverse of elasticity of labor supply	0.5	
	(II)		
ν	Weight of tradables in consumption	0.31	Johnson (2017)
ν_I	Weight of tradables in investment	0.40	Bems (2008)
α	Capital intensity in production	0.41	BEA
θ-1	Elasticity of trade	4.1	Simonovska and Waugh (2014)
ϕ	Elasticity of migration	4.5	Artuc et al. (2010)
	(III)		
ρ	Persistence matrix of productivity	$\begin{bmatrix} 0.65 & 0.06 \\ 0.04 & 0.53 \end{bmatrix}$	Estimated from GA and OH's TFP
Σ	Covariance matrix of shocks	$\begin{bmatrix} 1.21 & 1.25 \\ 1.25 & 2.56 \end{bmatrix} e^{-4}$	
	(IV)		
τ	Trade cost	1.031	Calibrated to match GA and OH's mean
d	Migration cost	19.58	export-to-output, emigrant-to-population,
f	Financial cost	3e-5	and consumption comovement

Under the specified parametrization, table 6 compares the contemporaneous correlations of variables in the calibrated model with those in the data. Panel (I) reports the cross-state comovement of output and consumption. The model performs well in matching empirical moments at both the aggregate and the per capita levels. In either case, output exhibits stronger cross-state synchronization than consumption. This result, which verifies the consumption correlation puzzle in the empirical section, points to the existence of frictions that impair consumption risk sharing. Panel (II) presents the correlation between a state's consumption with its own output per capita. Based on the finding that the correlation is greater than 0.9 in both the model and the data, consumption is highly procyclical. Furthermore, Panel (II) reports the correlation between a state's scaled net export (NX/Y) and population (N) with its own output (Y). Scaled

[7] Appendix C provides the technical details. The coefficient matrices include R_1, R_2, D_1, and D_2 in equations A16 and A17, which capture the responses of consumption differentials and excess asset returns to excess portfolio returns.

Table 6: Contemporaneous Correlations of Variables

	Model	Data
	(I) Cross-state Correlation	
Output $\rho(Y_1, Y_2)$	0.85	0.84
Consumption $\rho(C_1, C_2)$	0.79	0.78
Output per capita $\rho(y_1, y_2)$	0.84	0.88
Consumption per capita $\rho(c_1, c_2)$	0.82	0.82
	(II) Correlation with Self Output	
Consumption per capita $\rho(c, y)$	0.95	0.91
Net exports $\rho(NX/Y, Y)$	-0.04	-0.03
Population $\rho(N, Y)$	-0.01	-0.02

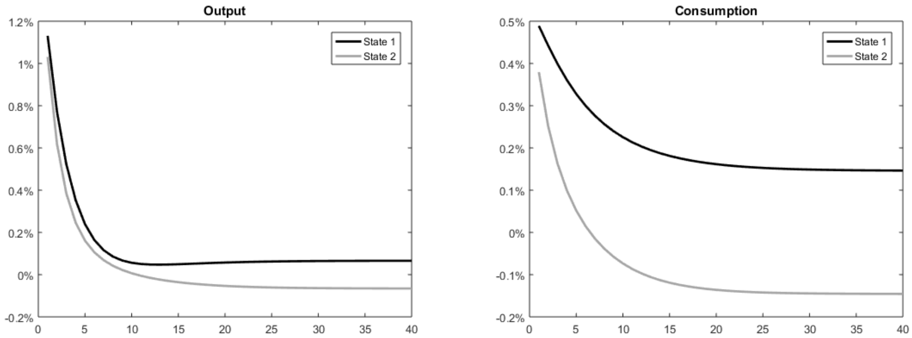
This table reports the contemporaneous correlations of Hodrick-Prescott filtered data and those in the calibrated model. Panel (I) reports the cross-region comovement of output and consumption at the aggregate (denoted as Y_i, C_i) and per capita (denoted as y_i, c_i) levels. Panel (II) reports the comovement of a region's scaled net exports (NX/Y) and population (N) with its own output, as well as the correlation between its consumption and output per capita.

net exports, measured as the ratio of the differences between exports and imports to output, turn out to be countercyclical. This finding is consistent with the international stylized facts documented by Mendoza (1991) and Backus et al. (1992). In addition, the contemporaneous correlation between population and output is negative both empirically and theoretically. Nevertheless, this correlation does not reflect the cumulative effects caused by delayed migration decisions under migration costs. To overcome such limitations, we examine the dynamic response of variables by plotting impulse response functions in the next step.

Figure 2 shows the impulse responses of the two states' output and consumption to a one-standard-deviation innovation in state 1's productivity. The black line shows the dynamics of state 1's variables and the grey line shows state 2's. Both states experience increases in output and consumption right after the productivity shock takes place. Even though the shock happens to state 1, there is positive spillover to state 2 not only due to productivity covariances but also thanks to cross-state goods, financial, and labor flows. Nevertheless, synchronization across states is not perfect and therefore state 1 witnesses greater improvements in its output and consumption.

To further understand the driving forces of synchronization, we examine the key variables of interest in the three channels. Figure 3 plots the impulse responses of state 1's terms of trade, external wealth position, investment, and population. Following a positive productivity shock to state 1, state 1 experiences a terms-of-trade depreciation as its exports become cheaper relative to imports to clear the goods market. This depreciation

Figure 2: Cross-state Comparison of Impulse Response Functions



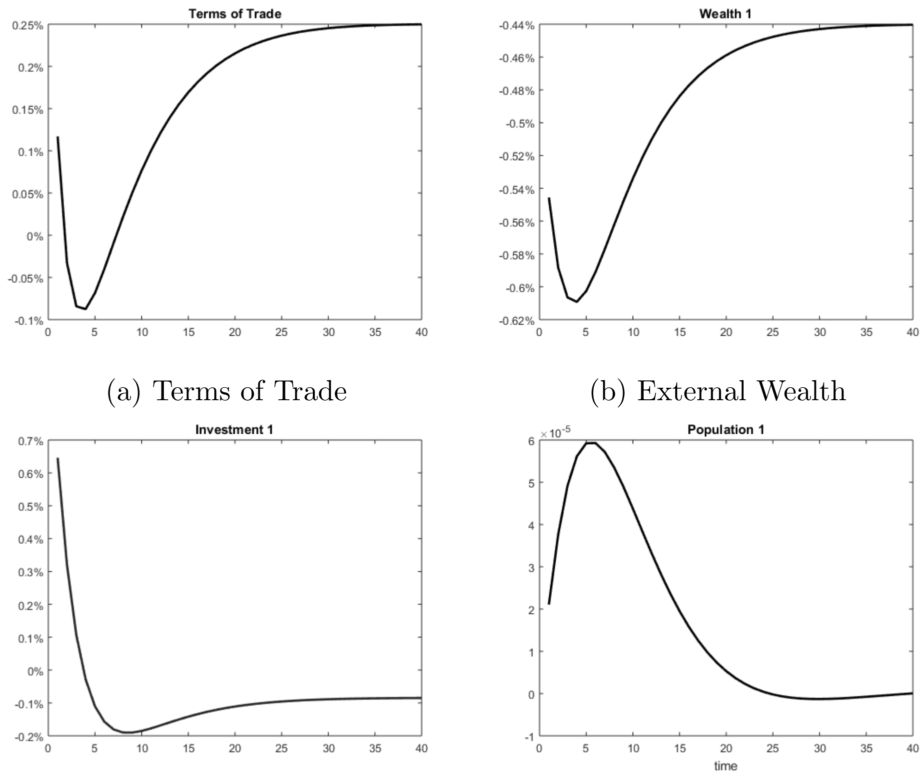
(a) Output (b) Consumption

Note: This figure plots the dynamic responses of macroeconomic variables, including output and consumption, to a one-standard-deviation innovation in state 1's productivity. The black and grey lines respectively show state 1's and state 2's variables. These variables are measured as a percentage of steady-state output in the plots.

will help increase the consumption of state 2, which does not experience the productivity boost, by raising its relative nominal income. Meanwhile, state 1 has a negative external wealth position, which suggests that it borrows from state 2. This could be understood from the fact that capital resources are allocated to the more productive economy where returns to investment are higher, which causes state 1's investment spike shown in figure 3c. As is argued by Heathcote and Perri (2013), this cross-border investment financing facilitates risk sharing. Lastly, population flows into state 1 (figure 3d), which raises the number of households among whom the increased aggregate consumption is shared and therefore helps to equalize consumption per capita across states. These quantitative results are mostly consistent with the qualitative analysis based on equation 37, with the exception that endogenous capital accumulation, which is absent from the qualitative analysis, alters the direction of financial flows in the short run, the same prediction as the one from the international RBC framework by Backus et al. (1992).

In the next step we conduct comparative analyses by varying the magnitude of the frictions to understand the impacts of barriers on the effectiveness of as well as the interactions among the channels of consumption risk sharing. Figures 4-6 illustrate the scenarios in which one type of friction doubles its calibrated value while the other parameters remain unchanged as in the baseline case. In the trade channel, state 1's terms-of-trade and exports to state 2 are less volatile when trade costs are high, as is shown in figure 4. This finding suggests that higher trade costs mute trade adjustments to productivity innovations, which leaves state 2 less benefited from state 1's positive productivity

Figure 3: Impulse Response of State 1's Macroeconomic Variables



(a) Terms of Trade

(b) External Wealth

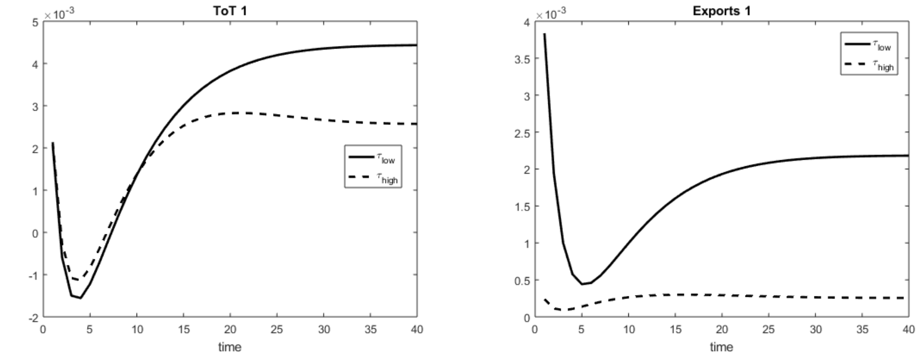
(c) Investment

(d) Population

Note: This figure plots the dynamic responses of macroeconomic variables to a one-standard-deviation innovation in state 1's productivity. Variables under examination include state 1's terms of trade, external wealth, investment, and population. They are measured as a percentage of steady-state output in the plots.

shock. In the financial channel (see figure 5), financial frictions raise state 1's cost of holding state 2's assets and generate asset home bias. However, the current dividends to assets, calculated as the difference between capital income and investment expenditure, are lower for state 1's assets than for state 2's given state 1's investment spike driven by the productivity shock. Therefore, in figure 5a higher financial frictions lower state 1's wealth accumulation by tilting portfolios toward temporarily lower-yielding domestic assets. Meanwhile, the financial channel has spillover effects on the migration channel by altering households' migration decisions. Lower financial frictions facilitate consumption risk sharing by allowing states to hold each others' assets, which dampens households' incentive to physically move across states in pursuit of higher consumption. Therefore, the dynamics of population are less volatile when financial frictions are lower in figure 5b.

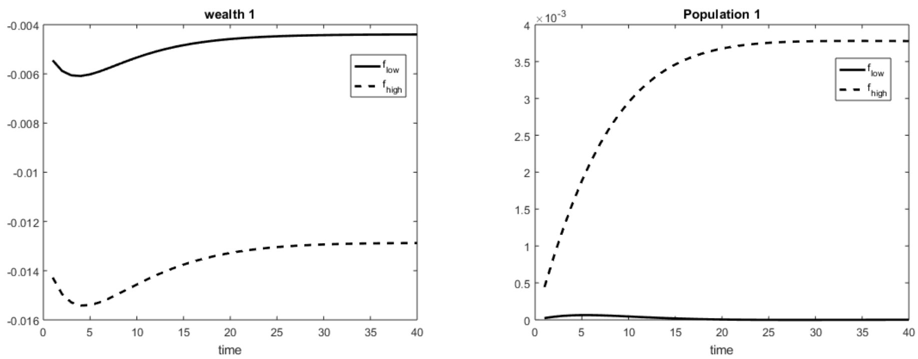
Figure 4: Comparative Analysis under Different Trade Costs



(a) Terms of Trade (b) Exports

Note: This figure plots the dynamic responses of state 1's terms of trade and exports to state 2, to a one-standard-deviation innovation in state 1's productivity. The solid and dashed lines show the situations with low and high trade costs respectively.

Figure 5: Comparative Analysis under Different Financial Frictions



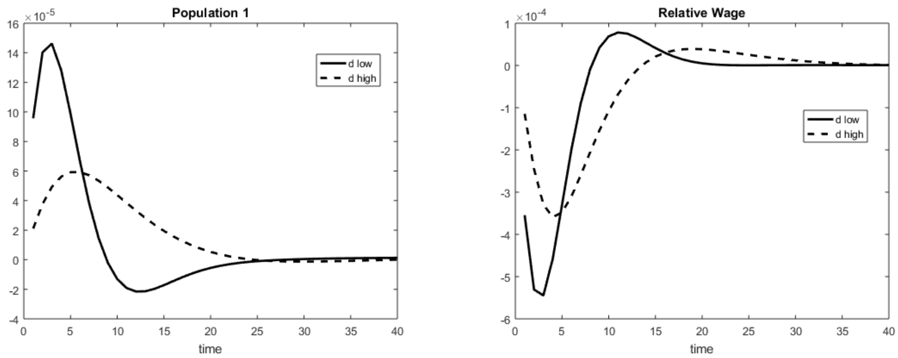
(a) Wealth (b) Population

Note: This figure plots the dynamic responses of state 1's external wealth and population, to a one-standard-deviation innovation in state 1's productivity. The solid and dashed lines show the situations with low and high financial frictions respectively.

The response of migration is also smaller when migration costs are higher, as is shown in figure 6a that raising migration costs flattens the curve of cross-state population flows. Under higher migration costs, not only is the magnitude of migration smaller, but also the duration of population flows is longer before reaching the new steady state. The hump-shaped migration pattern is driven by the forward-looking migration decisions subject to migration frictions. Moreover, the dynamics of the relative wage rate across states denoted as $w = \frac{w_1}{w_2}$ is depicted in figure 6b, which appears almost as a mirror image of figure

6a given the labor market clearing condition. The figure shows that higher migration costs cause smoother fluctuations in the relative wage. For example, the plunge of the relative wage right after state 1's positive productivity shock is larger when migration costs are lower. To understand this result, w_1 falls more relative to w_2 due to the terms-of-trade depreciation that reduces state 1's nominal marginal product of labor. If migration were to take place that drew more population to state 1 in response to its higher consumption growth, w_1 would decline even further to clear the labor market. Therefore, higher migration costs avoid a greater plummet in the relative wage and therefore increase wage synchronization across states.

Figure 6: Comparative Analysis under Different Migration Costs

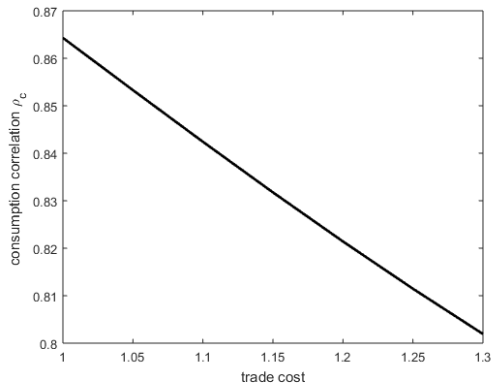


(a) Population (b) Relative wage

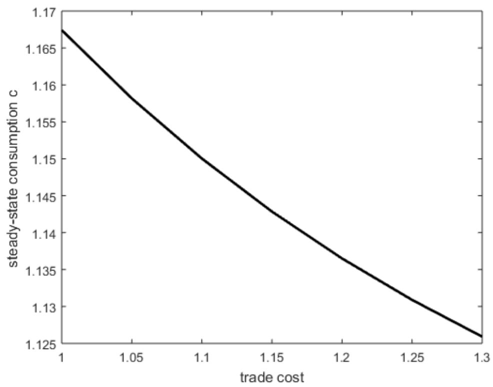
Note: This figure plots the dynamic responses of state 1's population and relative wage $w = \frac{w_1}{w_2}$, to a one-standard-deviation innovation in state 1's productivity. The solid and dashed lines show the situations with low and high migration costs respectively.

Based on these discussions, bilateral economic linkages through trade, migration, and finance, affect consumption risk sharing across regions. As a result, frictions in these channels have important implications for cross-state consumption comovement. We conduct another set of comparative analyses to test this hypothesis. Specifically, we calculate the model-predicted consumption correlation when changing the counterfactual value of one friction at a time. This exercise involves three steps. Step 1, we calculate the equilibrium values of all the variables on the real side of the economy under specific trade and migration frictions. Step 2, we solve the portfolio choice problem under financial frictions by evaluating the first- and second-order dynamics of the model. Step 3, we simulate the model that encompasses both real and financial allocations of the two-state economy and compute the resulting bilateral consumption comovement in all these counterfactual scenarios.

Figure 7: Consumption under Different Trade Costs



(a) $\rho(c_1, c_2)$

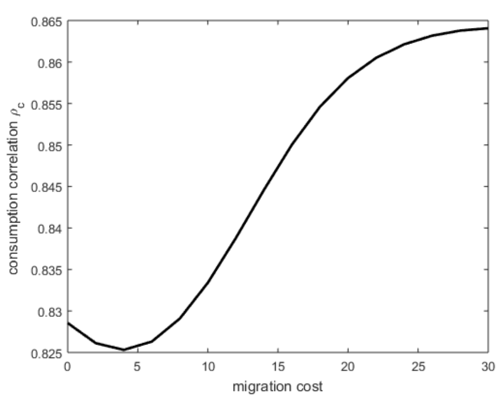


(b) $\bar{c}_i$

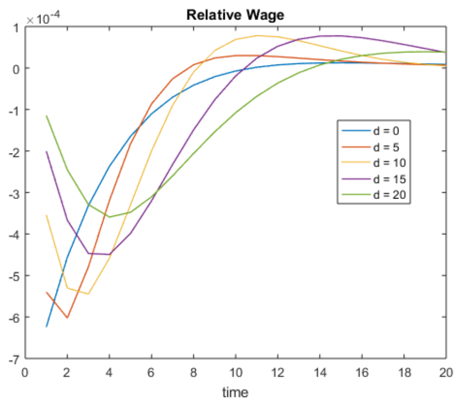
Note: This figure plots the pattern of consumption per capita under different trade costs. Figure 7a plots the correlation coefficient of consumption per capita across states given different trade costs. Figure 7b plots the state-level consumption per capita in the steady state of the economy.

Figure 7 shows the pattern of consumption per capita under different trade costs. Figure 7a plots the correlation coefficient of consumption per capita across states $\rho(c_1, c_2)$. The figure suggests that higher trade costs hinder cross-state consumption comovement. For example, when the trade cost is 1.3 consumption correlation is 0.802, which rises to 0.864 when there is no trade cost ($t = 1$). Besides raising the correlation coefficient as a second-moment variable, lowering trade costs also raises the level of consumption as a first-moment variable. Figure 7b illustrates the state-level consumption per capita in the steady state of the economy. The level of consumption increases from 1.13 to 1.17 when the trade cost decreases from 1.3 to 1, which is caused by the smaller loss of tradable goods during transportation under lower iceberg trade costs. Based on these findings, eliminating trade costs raises state-level consumption and facilitates cross-state risk sharing.

Figure 8: Consumption under Different Migration Costs



(a) $\rho(c_1, c_2)$

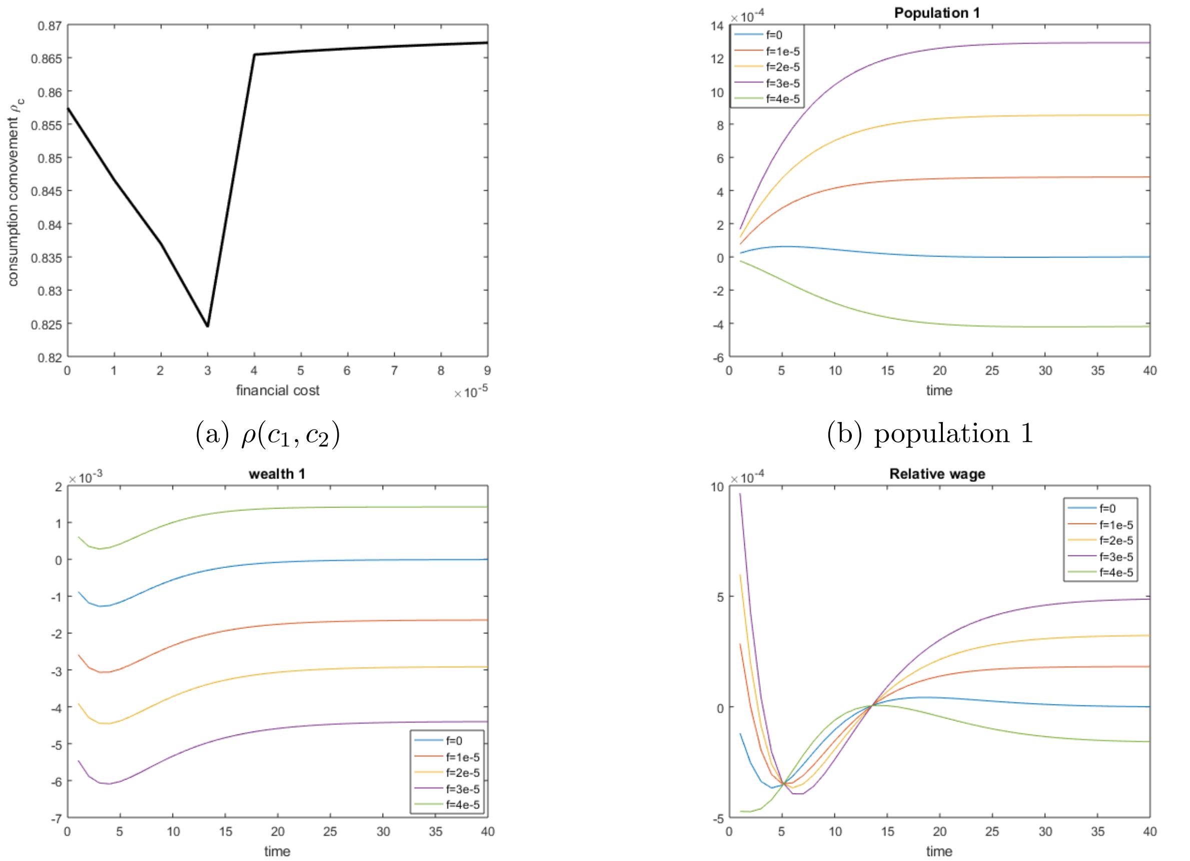


(b) Impulse response of w

Note: Figure 8a plots the correlation coefficient of consumption per capita across states given different migration costs. Figure 8b plots the impulse response of the cross-state wage ratio $w = \frac{w_1}{w_2}$ to a one-standard-deviation innovation in state 1's productivity.

Figure 8 shows the pattern of consumption per capita under different migration costs. The cross-state consumption correlation shown in figure 8a does not change monotonically with migration costs as with trade costs. When the costs first emerge around the neighborhood of zero, consumption comovement decreases, which suggests that higher migration costs impair consumption risk sharing. After that, consumption correlation increases with migration costs, although the concave curve suggests diminishing marginal effects of the migration costs. The shape of the curve is largely driven by the impact of the migration costs on the relative wage across states (denoted as $w = \frac{w_1}{w_2}$), whose impulse responses are plotted in figure 8b. Consistent with the earlier analysis for figure 6b, higher migration costs reduce the dynamics of the wage ratio through flattening population flows over time. A smoother relative wage pattern suggests a greater correlation of wage rates across states, which leads to a higher degree of consumption comovement since labor income is an important funding source for households' consumption expenditure. This explains the reason that higher migration costs raise the correlation coefficient of consumption in general (figure 6a). The exception to this general pattern happens when migration costs are too low to generate a smooth cross-state wage convergence (shown as the kink of the red line in figure 6b). Under this circumstance, consumption comovement deteriorates under higher migration costs.

Figure 9: Consumption under Different Financial Frictions



(a) $\rho(c_1, c_2)$

(b) population 1

(c) wealth 1

(d) relative w

Note: Figure 9a plots the correlation coefficient of consumption per capita across states given different financial frictions. Figures 9b-9d plot the impulse responses of state 1's population, wealth, and cross-state wage ratio $w = \frac{w_1}{w_2}$ to a one-standard-deviation innovation in state 1's productivity.

Figure 9 explores the patterns and determinants of consumption comovement under different financial frictions. As is shown in 9a, consumption correlation does not vary monotonically or smoothly with financial costs. Around the neighborhood of the calibrated financial friction $[0, 3]e^{-5}$, higher financial frictions lead to weaker consumption comovement. This is consistent with our analysis earlier: financial frictions raise the cost of holding foreign assets and tilt portfolios more toward domestic assets. Consequently, each state's consumption, driven more by its own output performance, is less synchronized with each other. What causes the discontinuity in figure 9a is the drastic change in the migration pattern shown in 9b. To understand this result, recall from the analysis for figure 5 that higher financial costs reduce state 1's wealth accumulation in response to its positive productivity shock (figure 9c). When financial frictions are sufficiently large ($\geq 4e^{-5}$), the deterioration of state 1's wealth position starts to negatively affect its consumption and hence alters the direction of population flows so that population moves from state 1 to 2 instead. Based on the same analysis as for figure 6b, this migration from state 1 to 2, by raising the relative wage of state 1, counteracts the impact of terms-of-trade depreciation after state 1's productivity shock. Therefore, the wage ratio across states is less volatile (figure 9d), which suggests that cross-state wage comovement is stronger. Given the importance of labor income for consumption expenditure, consumption correlation is stronger when financial costs are high enough to shift migration. Therefore, the three channels of consumption risk sharing are all manifested in figure 9a where they jointly shape the pattern of consumption correlation under financial frictions. This analysis underscores the importance of examining the interaction of these channels in the general equilibrium.

To conclude this section, we develop a theoretical model in which cross-region consumption synchronization is shaped by three channels: migration, trade, and finance. We use a two-region example calibrated to the US data to elucidate the interplay among the three channels and their joint effects on the consumption pattern. In the next section we extend the two-region case to a multi-region scenario to conduct a more comprehensive and realistic quantitative analysis of the theoretical model.

4 Quantitative Assessment

This section evaluates the theoretical model quantitatively in a multi-region DSGE framework, which allows us to quantify and disentangle the impacts of various frictions on spatial consumption comovement through counterfactual analyses. Furthermore, we use this general equilibrium model to deliver implications for macroeconomic policies that aim to improve welfare by raising consumption levels and reducing consumption fluctuations.

4.1 Extended Model

We enrich the framework for quantitative analysis in section 3.2 by relaxing the symmetric two-region assumption. First, the equilibrium population size is different across states and taken from their values from the data averaged over the sample period (1997-2017). Second, we extend the two-region to a multi-region case so that multilateral economic exchanges clear the goods, labor, and financial markets in aggregate. This extension allows us to examine the total effects of bilateral economic linkages on each region's macroeconomic variables.

Calibrated to the U.S. state-level data, the model encompasses $\mathcal{I} = 50$ regions. Ideally, a household in region i considers all the $\mathcal{I}$ regions when making economic decisions. One computational challenge we face when solving the multi-region DSGE model is that the large matrix that covers the bilateral ties for all the regions is badly scaled given the uneven distribution of economic sizes. Therefore, using this matrix to derive portfolio choice with the perturbation method yields inaccurate results. To overcome this challenge, we propose a trilateral framework when analyzing any region-pair formed by regions $i, j \in \mathcal{I}$. The framework consists of i and j, as well as the rest of the economy from i and j's perspective (ROE for simplicity). This trilateral framework not only enables the examination of bilateral frictions between i and j, but also considers the impacts of multilateral resistance of all the other regions affecting the region-pair. The latter echoes the extended gravity model in the trade literature developed by Anderson and Van Wincoop (2003) to capture the substitutability across trade partners.

In terms of parametrization, many parameters take the same values from the existing literature as in the two-economy framework summarized in table 5. For state-specific parameters, we follow the same strategy as in section 3.2 with modifications tailored to the trilateral framework. For example, we follow the literature including Backus et al. (1992) and Corsetti et al. (2008) to characterize productivity as the Solow residual. The variables of ROE, denoted with asterisks below, will be the sum of all the $\mathcal{I}$ regions' variables minus i and j's. Therefore, ROE's productivity at time t is computed from

$$\begin{aligned}\log(A_t^{ij*}) &= \log(Y_t^{ij*}) - \alpha \log(K_t^{ij*}) - (1-\alpha)\log(L_t^{ij*}) \\ &\equiv \log(\sum_i^{\mathcal{I}} Y_{i,t} - Y_{i,t} - Y_{j,t}) - \alpha \log(\sum_i^{\mathcal{I}} K_{i,t} - K_{i,t} - K_{j,t}) \\ &- (1-\alpha)\log(\sum_i^{\mathcal{I}} L_{i,t} - L_{i,t} - L_{j,t}).\end{aligned} \tag{39}$$

After that we obtain the variance-covariance matrix of these three regions' productivity assuming the annual persistence of productivity is 0.72, which is estimated from the U.S. national-level macro data. This assumption allows a state to have consistent productivity persistence among all the state pairs it forms. We estimate the variance-covariance matrix for all the $\frac{1}{2}\frac{\mathcal{I}}{\mathcal{I}-1} = 1225$ state pairs in the sample.

Another distinct feature of this asymmetric framework is that each region may not run a balanced budget in the equilibrium. To this end, we collect the data on state-level output and expenditure (defined as the sum of consumption and investment), whose difference represents the net asset position of the economy. ROE's asset position will be the sum of all the states' positions minus the positions of the state-pair under examination. The time-averaged asset positions will be reflected in portfolio choice.

We now proceed to discuss the calibration strategies for bilateral frictions in the trilateral framework. There are three economies numbered $1, 2, 3$ with 1 and 2 representing the pair of states under examination and 3 representing ROE. The three economies encounter a set of six bilateral frictions in each of the trade, migration, and finance channels

$$\{x_{12}, x_{13}, x_{23}, x_{21}, x_{31}, x_{32}\}, \qquad x \in \{\tau, d, f\}. \tag{40}$$

In terms of trade and migration costs, we estimate them simultaneously to ensure that the model-predicted bilateral migration and trade ties match those from the IRS and CFS data (see Appendix B for data description). The estimation procedure is similar to that in section 3.2: Step 1, we start with an initial guess for the combination of migration and trade costs. Step 2, we solve for wage rates and labor hours given the frictions that satisfy the labor market clearing condition (equation 9). Step 3, we calculate the corresponding bilateral trade shares ($\pi_{ij,t}$ in equation 29) and migration shares ($m_{ij,t}$ in equation 13) to the wages solved earlier. Step 4, we repeat the previous steps until the trade and migration shares converge to the empirical moments.

After characterizing the real side of the model, we estimate frictions in the financial channel. Due to the lack of state-to-state financial data, we estimate bilateral financial frictions indirectly from the pattern of consumption comovement across economies. For this purpose, we estimate the coefficients of consumption risk sharing with the same data and method as in the empirical section

$$\beta = [\beta_{12}, \beta_{13}, \beta_{23}], \tag{41}$$

and use the coefficients as targeted moments to estimate bilateral frictions. Appendix C outlines the technical details of the portfolio choice problem in this trilateral framework. The algorithm is slightly modified from that in section 3.2: First, we obtain the coefficient matrices, including R_1, R_2, D_1, D_2 in equations A16-A17, necessary to solve the portfolio choice problem from the first-order dynamics of the model. Second, we solve for asset holdings under which the model-implied risk-sharing coefficients β match those estimated from the data. To simplify our computation in this step, we assume a state's holding of ROE's assets is the same whose baseline weight in the portfolio is one-half but the state can choose the remaining composition between its own and pair partner's assets under risk-sharing motives. Third, we plug the calibrated asset positions in the portfolio determination equation (equation 24) to compute financial frictions.

Our benchmark calibration is based on the data over the sample time from 1997 to 2017. The sample selection is largely driven by the availability of the CFS trade data. We use the time-averaged state-level population, total asset positions, trade, and migration data over the sample period as the steady-state values of those variables when estimating and solving the model. We evaluate the model fit by comparing empirical and model-predicted moments of bilateral variables. Figure A.2 presents the performance of the model in matching targeted moments including bilateral trade shares, bilateral migration shares, and coefficients of consumption risk sharing. From the figures, the model does a good job matching these empirical moments since most of the observations fall on the 45-degree line. In terms of untargeted moments, the key variable of interest is bilateral consumption correlation. To obtain its predicted value from the quantitative model, first we compute the steady-state values of all the endogenous variables in this DSGE model after calibrating the productivity shocks and bilateral frictions that generate consistent moments with the data. Then we simulate the model with productivity shocks and examine the impulse responses of consumption per capita in different states. Lastly, we compute bilateral consumption correlation, averaged over the simulated shocks, and compare it to the counterpart from the empirical section. Figure A.3 suggests that the model does modestly well in predicting consumption comovement across all the state pairs. In Wyoming's case, the model successfully predicts that Wyoming exhibits a notably stronger consumption correlation with Texas and West Virginia than Massachusetts and Minnesota.

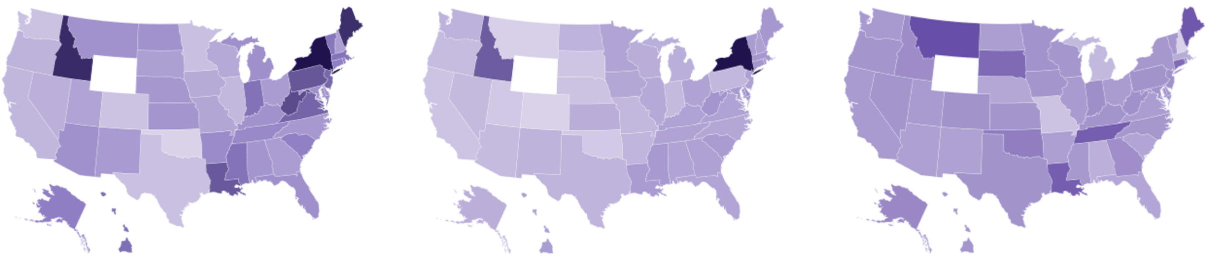
After evaluating the model performance in matching targeted and untargeted moments, we now proceed to discuss numerical predictions from the quantitative model.

4.2 Numerical Results

This section presents the predictions of the quantitative model. First, we estimate the magnitude of frictions estimated from the model. Second, we conduct a set of counterfactual analyses to quantify the effects of each friction. Lastly, we solve for optimal macroeconomic policies which, by offsetting the impacts of frictions on consumption, could improve social welfare.

To provide a first glance of the frictions in the three channels of risk sharing, we use Wyoming (WY) as an example by showing the heatmaps of its estimated bilateral frictions with other states in figure 10. Each type of bilateral friction is calculated as the geometric mean of outbound and inbound frictions ($x_{WY,i},\ x_{i,WY},\ i \in [1, \mathcal{I}], x \in \{\tau, d, f\}$) between Wyoming (in white) and any other state. In general, states located within a smaller radius from Wyoming exhibit lower frictions with the state. For example, the migration cost between Wyoming and a neighboring state Colorado is the lowest, whose value is approximately 1/3 of that between Wyoming and Hawaii. This spatial pattern is consistent with the observation in figure 1 that Wyoming shows stronger economic ties with states which are geographically closer. However, there are exceptions to the pattern. Idaho, another neighboring state of Wyoming, is estimated to inflict high trade costs due to its low trade volume with Wyoming unexplained by the size of its aggregate expenditure.

Figure 10: Wyoming's Estimated Frictions with Other States



(a) Trade (b) Migration (c) Finance

This figure plots the estimated bilateral frictions between Wyoming (in white) and other states in the U.S. A darker color suggests a higher value of friction. Frictions are calculated as the geometric average of bidirectional frictions.

To explore the geographic characteristics of frictions in general, we run bivariate regressions with the estimated bidirectional frictions as dependent variables and geographic distance as the independent variable for all the $\frac{\mathcal{I}(\mathcal{I}-1)}{2}$ state pairs:

$$\log(\hat{x}_{ij}) = \alpha_x + \gamma_x \log(dist_{ij}) + \epsilon_{ij}, \qquad x \in \{\tau, d, f\}. \tag{42}$$

As reported in table 7, a 1% rise in distance is associated with a 0.525% increase in trade costs, a 0.100% increase in migration costs, and a 0.232% increase in financial frictions. By comparing these values, we infer that trade costs are most sensitive to distance. All the coefficient estimates ($\hat{\gamma}_x$ in regression 42) are significantly positive. This numerical

Table 7: Bilateral frictions and Geographic Distance

Dep. Var: Est. Frictions	$\log(\hat{\tau_{ij}})$	$\log(\hat{d}_{ij})$	$\log(\hat{f}_{ij})$
$\log(dist_{ij})$	0.525 ***	0.100 ***	0.232 **
	(0.047)	(0.01)	0.097
Observations	2442	2442	2442
R^2	0.041	0.023	0.003

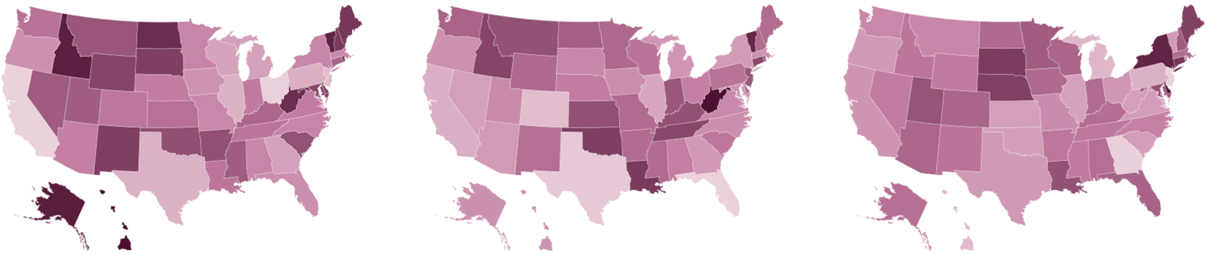
This table reports the regression results of equation 42. Robust standard errors in parentheses, standardized coefficients in brackets. *** significant at 1%, ** significant at 5%. Estimated frictions are missing for some pairs because the eigenvalues computed at the steady state of the model for those pairs do not satisfy the Blanchard and Kahn (1980) condition to guarantee the existence of a unique solution.

result confirms one of the key hypotheses of this paper that frictions that impair risk sharing covary with geographic distance between states, which potentially shapes the spatial pattern of consumption.

To evaluate and compare frictions at the state level, we compute the median frictions across all the state-pairs each state forms and present them in figure 11 and table A.4. Based on the estimation results, the trade costs of Hawaii and Alaska, the two non-contiguous states, are among the highest. For example, the median outbound trade cost of Alaska is 3.14 times the cost of Georgia and Ohio, the median states in terms of output per capita. In the migration channel, Florida and Texas are estimated to face the lowest inbound migration costs. This finding coincides with the observation that these two states are popular destinations of migration inflows in recent decades. In the financial channel, states whose estimated financial frictions are the highest include Delaware, Alaska, and Nebraska. This result is largely driven by the relatively low degree of consumption risk sharing unexplained by trade and migration channels between these states and others.

After discussing the magnitude of estimated frictions, we proceed to quantify their impacts by conducting counterfactual analyses where we turn off one friction at a time. To focus on the impacts of bilateral frictions, we will first shut down the frictions between a pair of states denoted as economies 1 and 2 ($x_{12}, x_{21}, x \in \{\tau, d, f\}$) but not the frictions between either state with ROE denoted as economy 3 ($x_{i3}, x_{3i}, i \in \{1, 2\}, x \in \{\tau, d, f\}$). This set of counterfactual analyses consists of three parts. First, we examine the influences of bilateral frictions on bilateral ties in the three channels of risk sharing. Second, we evaluate the effects of frictions on bilateral consumption comovement. Third, we compute the state-level consumption level and volatility averaged across pairs to study the overall effects of these frictions.

Figure 11: Average Friction by State



(a) Trade (b) Migration (c) Finance

This figure plots the estimated frictions averaged across all the state-pairs each state forms. A darker color suggests a higher value of frictions. Frictions are calculated as the geometric average of bidirectional frictions in logarithms.

Table 8 reports the key statistics of bilateral trade, migration, and asset shares across all the state pairs in the sample. If we use our earlier notations by denoting the three economies when analyzing a state-pair as 1,2, and 3 with 3 representing the rest of the economy (ROE), and denoting exports from i to j as X_{ij}, average bilateral trade shares between 1 and 2 equal

$$\frac{1}{2}(\pi_{12}+\pi_{21}) = \frac{1}{2}(\frac{X_{12}}{\sum_{i=1,2,3} X_{1i}} + \frac{X_{21}}{\sum_{i=1,2,3} X_{2i}}). \tag{43}$$

Similarly, with population flows from i to j denoted as N_{ij}, we calculate bilateral migration shares (m) as

$$\frac{1}{2}(m_{12}+m_{21}) = \frac{1}{2}(\frac{N_{12}}{\sum_{i=1,2,3} N_{1i}} + \frac{N_{21}}{\sum_{i=1,2,3} N_{2i}}). \tag{44}$$

With j's holding of asset from i denoted as α_{ij}, bilateral financial shares $(\hat{\alpha})$ can be computed as

$$\frac{1}{2}(\hat{\alpha}_{12}+\hat{\alpha}_{21}) = \frac{1}{2}(\frac{\alpha_{12}}{\sum_{i=1,2,3} \alpha_{i2}} + \frac{\alpha_{21}}{\sum_{i=1,2,3} \alpha_{i1}}). \tag{45}$$

Following these formulas, we calculate these bilateral shares in the original case under the calibrated frictions and in the counterfactual case where corresponding bilateral frictions $x_{12}, x_{21}, x \in \{\tau, d, f\}$ are turned off. As is reported in table 8, bilateral economic ties in all the three channels strengthen remarkably under counterfactual scenarios absent bilateral frictions. For example, bilateral trade shares rise from 0.006 to 0.444 on average across state pairs when bilateral trade costs are assumed to be 1. Moreover, the elimination of migration costs raises average bilateral migration shares from 0.0007 to 0.4914, while

Table 8: Counterfactual Bilateral Ties

	(I). With Friction		(II). Without Friction	
	Mean	Median	Mean	Median
Trade	0.0060	0.0029	0.4440	0.4548
Migration	0.0007	0.0004	0.4914	0.4918
Finance	0.1636	0.1557	0.2764	0.2466

This table reports the counterfactual bilateral exchanges when corresponding bilateral frictions are set to zero. Variables reported include the mean and median values of bilateral trade, migration, and asset shares across all the state pairs in the sample.

the elimination of financial frictions raises bilateral asset holdings from 0.1636 to 0.2764. What is common about these counterfactual scenarios is that, these bilateral shares turn out to be close in value to each state's own shares for trade, migration, and finance:

$$z_{12} \approx z_{11}, \qquad z \in \{\pi, m, \alpha\}. \tag{46}$$

The reasoning behind this result is that when a pair of states form an economic zone without barriers, they treat each other like themselves when exchanging goods, labor, and assets. Meanwhile, they drastically cut economic linkages from the rest of the economy with which frictions are considerably higher.

After evaluating the impacts of bilateral frictions on each channel of economic exchanges, we examine the counterfactual pattern of bilateral consumption comovement. Specifically, we focus on two measures: first, the bilateral correlation coefficient of consumption per capita (ρ_c) and second, the degree of consumption risk sharing ($\beta_c = 1-\beta$), measured as the difference between unity and the response of relative consumption growth to output growth between a pair of states. Table 9 reports the median values of these two variables across the state pairs in the sample. The median correlation of consumption increases from 0.4177 in the benchmark case with calibrated frictions to 0.7451 when there is no trade cost. Meanwhile, the correlation slightly drops to 0.4141 and 0.4064 when there is no migration cost and no financial friction respectively. These findings are largely consistent with the two-region analysis from the theory section (see figures 7-9). While the decrease in trade costs inarguably raises consumption correlations, the reduction in migration or financial frictions yields nonmonotonic predictions. In the situation where migration exacerbates cross-region wage inequality following terms-of-trade adjustments, a decline in the migration costs leads to lower consumption correlation. Moreover, when financial frictions are so low as to encourage migration in the direction that worsens wage

Table 9: Counterfactual Bilateral Consumption Comovement

	Org	No τ	No d	No f
ρ_c	0.4177	0.7451	0.4141	0.4064
β_c	0.5324	0.7396	0.8181	0.7097

This table reports the median bilateral consumption correlation (ρ_c) and degree of risk sharing (β_c) across all the state pairs in the original case and when trade costs (τ), migration costs (d), and financial frictions (f) are turned off.

disparity, raising the financial frictions helps improve consumption comovement across economies. It is worth noting that a decrease in consumption correlation may not necessarily mean a deterioration in consumption risk sharing, because cross-region output correlation may also vary with the reduction of bilateral frictions. To this end, we examine the degree of consumption risk sharing β_c. From table 9, the median value of β_c increases remarkably from 0.5324 in the original case to 0.7396, 0.8181, and 0.7097 in the counterfactual scenarios where trade, migration, and financial frictions are turned off respectively. This numerical result suggests that eliminating the frictions in these channels will reduce the impacts of local output shocks on consumption fluctuations by allowing economies to share risks with each other.

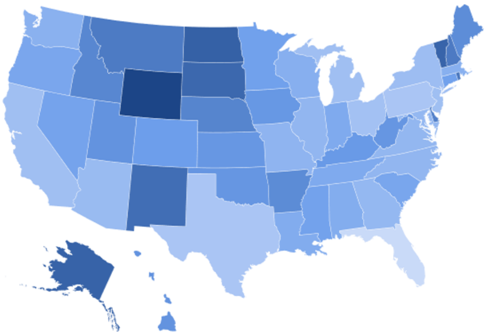
Furthermore, we examine the overall impacts of bilateral frictions on state-level consumption. For this purpose, we first compute the steady-state level as well as the volatility of consumption per capita of the two states forming each of the $\frac{\mathcal{I}(\mathcal{I}-1)}{2} = 1225$ state pairs. After that, we take the median value across the $\mathcal{I} - 1 = 49$ state pairs involving a specific state as that state's consumption level and volatility. Table A.5 presents the ratio of counterfactual consumption to that in the original case. Based on the reported values, figures 12-13a visualize the counterfactual consumption level compared to that in the original case.[8] Most states witness improvements to their consumption levels when there is no trade cost. In general, states that are subject to the highest trade costs experience the greatest boost in consumption under the counterfactual circumstance. For example, Alaska's consumption rises by 29.8% with the reduction of trade costs. Across the states, the median increase in consumption under the elimination of trade costs is 7.3%.

The reduction in migration costs, on the other hand, generates a more disparate pattern across states. While the most affluent states including Florida and California benefit

[8]Since financial frictions are second-order in magnitude and will therefore not affect the level of consumption in the steady state of the model, we focus our analysis here on the situations with no trade and migration costs.

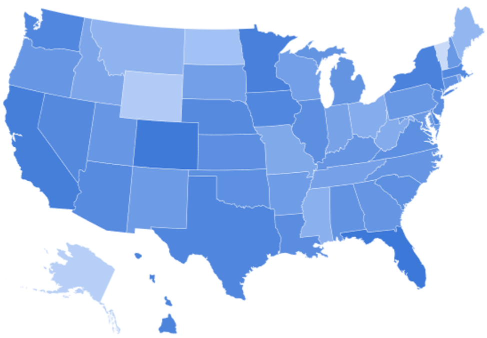
from labor mobility, most other states expect lower consumption per capita when the restriction on population is lifted. Across the states, the median change in consumption per capita is -3.2% when migration costs are removed. To explain this pattern, we show the change of each state's population size in figure 13b whose geographic pattern is almost the opposite to figure 13a's: a larger population is associated with a lower consumption per capita. What happens is that the elimination of bilateral migration costs causes drastic population inflows for most states from the rest of the economy (ROE). This is not driven by the change in the migration cost between a state-pair and ROE (i.e. $d_{i3}, d_{3i}, i = 1, 2$) which is assumed to be fixed in this counterfactual analysis, but driven by the change in the migration costs between the pair of states (i.e. d_{12}, d_{21}). Based on the rule of migration decisions (equation 27), households move to a state with a high "option value," which captures the expected future payoff from moving from that state to other states. Therefore, states like Wyoming with the darkest color in figure 13b attract large migration inflows because people find it easier to move from those states to California due to the reduction in their bilateral migration costs with California. Nevertheless, unlike California whose TFP is high enough to benefit from the rise in labor supply, Wyoming does not have enough jobs to meet the needs of the increased population. As a result, wage declines under the labor market clearing condition, which translates to a lower consumption level. This reasoning explains the disparate pattern of consumption across states generated by the reduction in migration costs in figure 13a. Nevertheless, it is worth noting that our current theoretical framework assumes that consumption is the main driver for migration and therefore neglects other factors including residential land and amenities which also play an important role in migration decisions (see, for example, Saiz (2010), Albouy and Lue (2015), Monte et al. (2018)). Once considered, these factors will dampen households' motives to migrate to states with less elastic housing supply and lower quality of amenities, and reshape the counterfactual population pattern in figure 13b. Despite these considerations, the elimination of migration costs still generates consumption gains for some states and losses for others, largely due to the zero-sum redistribution of population across states it causes.

Figure 12: Counterfactual Consumption without Trade Costs

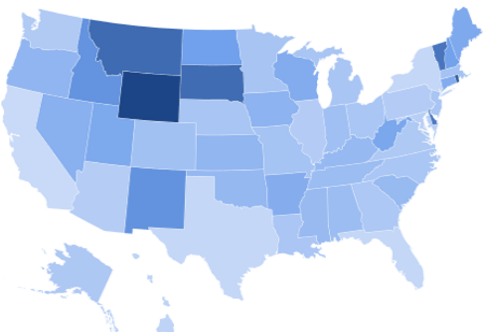


Note: This figure plots the ratio of counterfactual to original level of consumption per capita in the steady state of the economy when bilateral trade cost is shut down. A darker color in the map suggests a higher ratio. Data are reported in table A.5.

Figure 13: Counterfactual Consumption and Population without Migration Costs



(a) Consumption p.c.

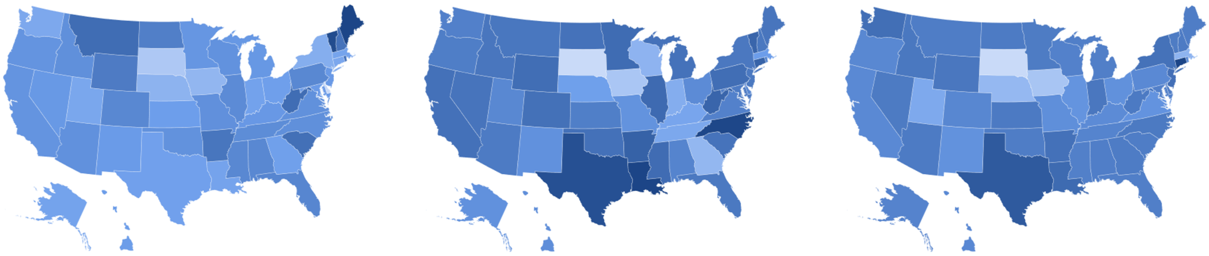


(b) Population

Note: This figure plots the ratio of counterfactual to original level of consumption per capita and population in the steady state of the economy when bilateral migration cost is shut down. A darker color in the map suggests a higher ratio. Data are reported in table A.5.

After examining the level of consumption per capita, we continue to investigate its volatility measured as the standard deviation. Figure 14 illustrates the ratio of consumption volatility in the counterfactual case to that in the original case. As is reported in table A.5, the volatility of consumption is lower on average under all the three counterfactual scenarios. The mean reduction in consumption volatility across states is 0.7%, 1.0%, and 0.3% respectively when bilateral trade, migration, and financial frictions are turned off. The magnitude of change is relatively small since it is driven by the elimination of bilateral frictions but not the overall frictions with respect to the rest of the economy. The three plots in figure 14 exhibit geographic resemblance, which implies the substitutability among the channels of risk sharing in lowering consumption volatility of the states most subject to frictions. For a risk-averse agent, lower consumption volatility indicates higher lifetime utility. Therefore, the finding that shutting down the frictions reduces consumption fluctuations reiterates the significance of the three channels of risk sharing for improving welfare.

Figure 14: Counterfactual Volatility of Consumption



(a) No τ (b) No d (c) No f

Note: This figure plots the ratio of counterfactual to original volatility of consumption per capita. A darker color suggests a higher ratio. Data are reported in table A.5.

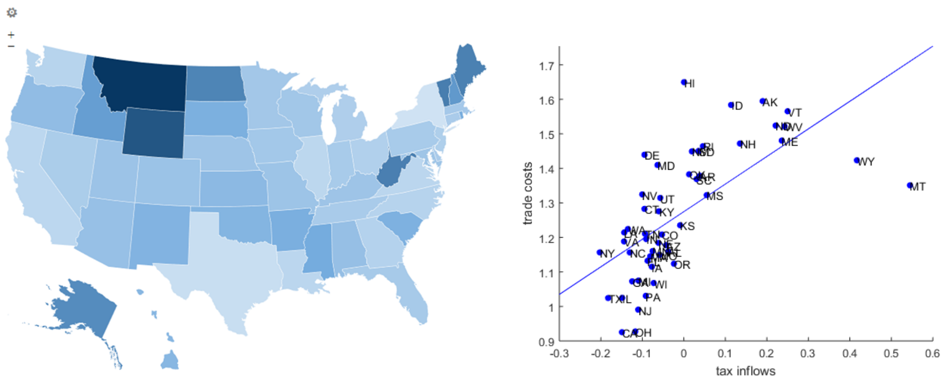
Last but not least, we use the counterfactual exercises conducted above to deliver policy implications. Our earlier analysis about the spatial characteristics of frictions indicates that eliminating the barriers in the channels of risk sharing can be challenging due to geographic constraints. Nevertheless, we can design macroeconomic policies to alleviate the negative impacts of the frictions. In particular, fiscal transfers have been acknowledged as an important channel of risk sharing within a country. Redistribution of wealth from beneficiaries to victims of frictions can potentially undo the influences of frictions on the level and volatility of consumption. On the modeling side, introducing fiscal transfers T_i rewrites the wealth constraint of state i

$$\mathcal{W}_{i,t+1} = R_{\mathcal{I},t}\mathcal{W}_{i,t} + \sum_{j}^{\mathcal{I}} \alpha_{j,i,t}(e^{-f_{ji}}R_{j,t} - e^{-f_{\mathcal{I}i}}R_{\mathcal{I},t}) + p_{i,t}\sum_{s} Y_{is,t} + T_{i,t} - P_{i,t}C_{i,t} - P_{Ii,t}I_{i,t}, \tag{47}$$

Under the new constraint, households in state i can adjust their expenditure on consumption and make necessary migration decisions based on the new cross-state consumption differentials. Meanwhile, the portfolio of state i can be re-constructed according to the risk-sharing needs under the new wealth constraint. Therefore, the design of fiscal policies requires taking into consideration the endogenous changes of variables in the existing channels of risk sharing and the interplay among these channels.

To exemplify such policy analysis, we examine the fiscal transfer that mitigates the reduction of consumption caused by bilateral trade costs. To keep this example simple, we don't impose an aggregate budget constraint for the federal government or restrict the amount of transfers it distributes to any state. The analysis involves the following steps. Step 1, we calculate the policy's targeted moment which is the counterfactual level of consumption when there is no bilateral trade cost. Step 2, we use the state-level wages in the original case as the initial guess, and solve for the supply and demand of labor in each state given the counterfactual trade and migration pattern under the new trade costs. After that, we update the value of wages that clear the labor market. We repeat this procedure until the difference between the old and new wages is small enough which pins down the equilibrium wages under the counterfactual scenario. Step 3, we solve for the values of all the endogenous variables in the model based on the wages from step 2 and calculate the corresponding level of consumption. Step 4, we repeat steps 2 and 3 until the model-predicted consumption converges to the targeted moment from step 1. We conduct this analysis for all the state pairs and, for cross-state comparison plot the median tax transfers across the state pairs formed by each state in figure 15. It illustrates that regions that are estimated to be confronted with higher trade costs, such as Wyoming, Montana, and Alaska, should receive more tax transfers to mitigate the impacts of trade frictions on their consumption. In contrast, states that face lower trade costs, including New York, Texas, and California, should be net tax payers to achieve the counterfactual outcome. The general relationship between the predicted transfers and the estimated trade costs is positive.

Figure 15: Tax Transfers under Trade Costs



(a) Predicted Transfer Inflows

(b) Estimated τ and Transfers

Note: This figure plots the tax transfers as shares of a state's GSP to achieve its level of consumption in the counterfactual situation absent trade costs. A darker color in the heatmap suggests more tax inflows. The scatter plot shows the positive relationship between the transfer and estimated trade costs (reported in table A.4).

This example shows that the quantitative model we propose in this paper provides a useful framework for policy analyses. The framework is general enough to accommodate a rich set of targeted moments including the level, volatility, and covariance of macroeconomic variables. Meanwhile, the framework is flexible enough to be adapted to any other country under the specific budget constraint its government is subject to. These policies, if well designed and implemented, facilitate risk sharing and hence reduce both consumption volatility over time and consumption disparity across regions.

5 Conclusion

This paper explores the role of bilateral economic exchanges influenced by geography in shaping the pattern of consumption comovement across 50 states in the US. Failure of consumption risk sharing has been recognized as a major puzzle in the macroeconomic literature. To explain this puzzle, our research exploits variations among state pairs and analyzes frictions that dampen bilateral consumption comovement. For this purpose, we propose a comprehensive and unified approach that encompasses trade, migration, and finance as channels of consumption risk sharing.

In the paper we first empirically establish a gravity model of consumption risk sharing by documenting that bilateral risk sharing decreases in geographic distance among the US states. To explain this fact, we develop a theoretical model to explore the impacts of frictions in the channels of risk sharing that potentially covary with distance. We start with a two-economy framework following Backus et al. (1992) to examine the mechanism of different channels affect consumption as well as how they interact with each other. After that, we extend the model to a multi-region framework calibrated to the US data for a quantitative assessment. The framework enables us to quantify not only the magnitude but also the influence of each friction through counterfactual analysis. The quantitative framework also serves as a useful tool for the design of macroeconomic policies which aim to reduce consumption disparity across time and space.

One important extension of our real business cycle (RBC) framework is to introduce the New Keynesian ingredients including nominal rigidity. As is pointed out by Hazell et al. (2022) that even within a monetary union, cross-region heterogeneity generates different slopes of the Phillips Curve under a uniform national monetary policy, which consequently creates welfare disparity across economies. Therefore, extensions of our model can incorporate monetary factors into the analysis when explaining cross-state consumption comovement. Meanwhile, the frictional economic linkages examined by our model, in particular through the channels of finance and migration, are absent in their analysis. Hence, our paper complements that literature by accounting for the transmission and prorogation of economic shocks through disaggregate cross-region economic ties. Other papers pursuing this research direction include House et al. (2018) and House et al. (2020), which quantify the welfare outcome of micro-founded economic ties under monetary policies.

Our paper focuses on cross-state risk sharing within the US as an example, but our theoretical framework is general and flexible enough to be tailored to another context of interest to examine bilateral linkages across economies through various channels. Therefore, we can apply the framework to explain consumption synchronization not only within but also across countries, or even both simultaneously. In particular, our framework is useful to be employed in such a context as the European Union and NAFTA where bilateral exchanges in different channels are commonplace. For example, one application of the model is to compare intra- versus inter-national linkages to diagnose the border effects of risk sharing proposed by Devereux and Hnatkovska (2020), who document a sharp decrease of consumption comovement at the US-Canada border as opposed to the prediction made by Backus and Smith (1993). By quantifying the magnitudes and impacts of frictions in channels of risk sharing in this setting, our framework can provide guidance for trade and exchange rate policies with a target to reduce consumption disparity and raise social welfare both within and across country borders.

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Figure A.1: U.S. Map

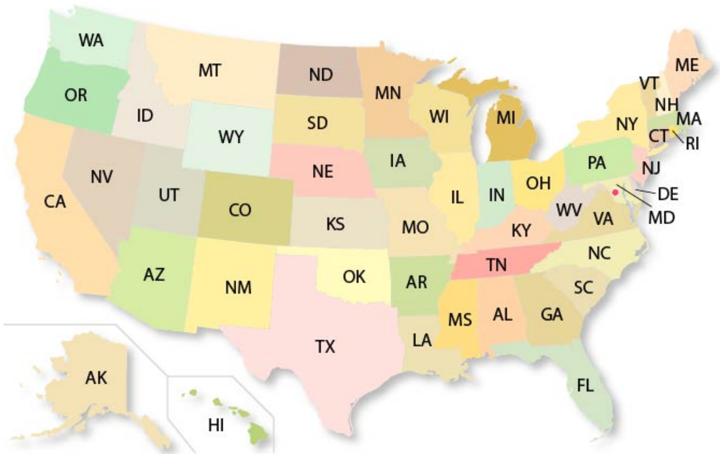


Table A.1: List of US States with Abbreviations

Name	Abbreviation	Name	Abbreviation	Name	Abbreviation	Name	Abbreviation	Name	Abbreviation
Alabama	AL	Hawaii	HI	Massachusetts	MA	New Mexico	NM	South Dakota	SD
Alaska	AK	Idaho	ID	Michigan	MI	New York	NY	Tennessee	TN
Arizona	AZ	Illinois	IL	Minnesota	MN	North Carolina	NC	Texas	TX
Arkansas	AR	Indiana	IN	Mississippi	MS	North Dakota	ND	Utah	UT
California	CA	Iowa	IA	Missouri	MO	Ohio	OH	Vermont	VT
Colorado	CO	Kansas	KS	Montana	MT	Oklahoma	OK	Virginia	VA
Connecticut	CT	Kentucky	KY	Nebraska	NE	Oregon	OR	Washington	WA
Delaware	DE	Louisiana	LA	Nevada	NV	Pennsylvania	PA	West Virginia	WV
Florida	FL	Maine	ME	New Hampshire	NH	Rhode Island	RI	Wisconsin	WI
Georgia	GA	Maryland	MD	New Jersey	NJ	South Carolina	SC	Wyoming	WY

Appendices

A Figures and Tables

Table A.3 (Panel A) reports the results of a robustness check of the gravity model of risk sharing where we adjust for states' distinct exposure to aggregate risks. In equation 1 where we define and estimate the risk-sharing coefficients, we use the difference in output growth between a pair of states to capture idiosyncratic risks. However, this difference can also reflect the two states' heterogeneous exposure to national shocks. To address this potential mis-measurement of local output shocks, we first estimate β_i and β_j from

$$\Delta \log\ y_{it} = \alpha_i + \beta_i \Delta \log\ y_{USt} + \epsilon_{it}, \qquad \Delta \log\ y_{jt} = \alpha_j + \beta_j \Delta \log\ y_{USt} + \epsilon_{jt}, \qquad \text{(A1)}$$

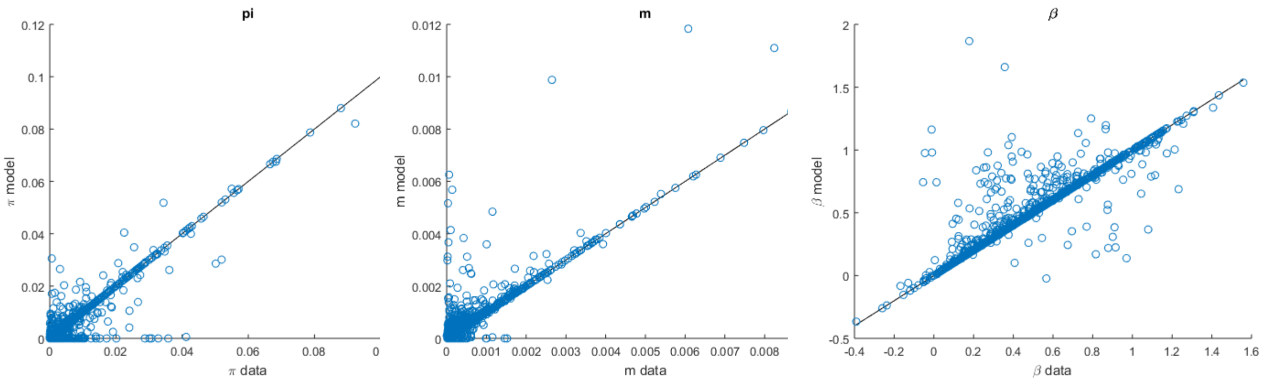
where $\Delta \log\ y_{USt}$ denotes the growth of log real per-capita output of the United States, and hence β_i captures the impact of aggregate shocks on state i's output. After that, we calculate bilateral risk-sharing coefficients from the response of consumption to a more robust measure of idiosyncratic output shocks when states' comovement with national

Table A.2: Gravity Model of Risk Sharing – Alternative Data Sources

Dep. Var.: $\hat{\beta}_{ij}$	A. CPI by Hazell et. al.			B. Consumption from BEA		
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(d_{ij})$	0.119 ***	0.123 ***	0.155 ***	0.041 ***	0.043 ***	0.049 ***
	(0.017)	(0.017)	(0.022)	(0.004)	(0.005)	(0.006)
$\log(\bar{y}_1 \cdot \bar{y}_2)$		-0.035	-0.160 **		-0.037 ***	-0.057 ***
		(0.064)	(0.074)		(0.013)	(0.015)
$\log(\sigma(y_1) \cdot \sigma(y_2))$			0.152 ***			0.032 ***
			(0.055)			(0.011)
Border			0.055			0.038 **
			(0.056)			(0.016)
$\log(\bar{N}_1 \cdot \bar{N}_2)$			0.024 ***			-0.013 ***
			(0.013)			(0.003)
Obs.	528	528	528	1225	1225	1225
R^2	0.077	0.077	0.102	0.056	0.061	0.090

Robust standard errors in parentheses. *** significant at 1%. The dependent variable is the risk sharing coefficient $\hat{\beta}_{ij}$, which is estimated using the real consumption and output data over 1977 – 2019. $\log d_{ij}$ denotes the geographic distance between state i and j in logarithms. y_i and N_i denote real output per capita and population in logarithms of state i. $\bar{x}$ and $\sigma(x)$ represent the mean and volatility of a variable x over the sample period.

Figure A.2: Model Fit (I)



Note: This figure plots the relationship between model-implied and actual bilateral ties including bilateral trade shares, bilateral migration shares, and coefficients of consumption risk sharing. Empirical moments are on the horizontal axis, and theoretical moments are on the vertical axis.

output is taken into consideration:

$$\Delta \log\ c_{it} - \Delta \log\ c_{jt} = \alpha_{ij} + \beta_{ij}[(\Delta \log\ y_{it} - \beta_i \Delta \log\ y_{USt}) - (\Delta \log\ y_{jt} - \beta_j \Delta \log\ y_{USt})] + \epsilon_{ijt}. \tag{A2}$$

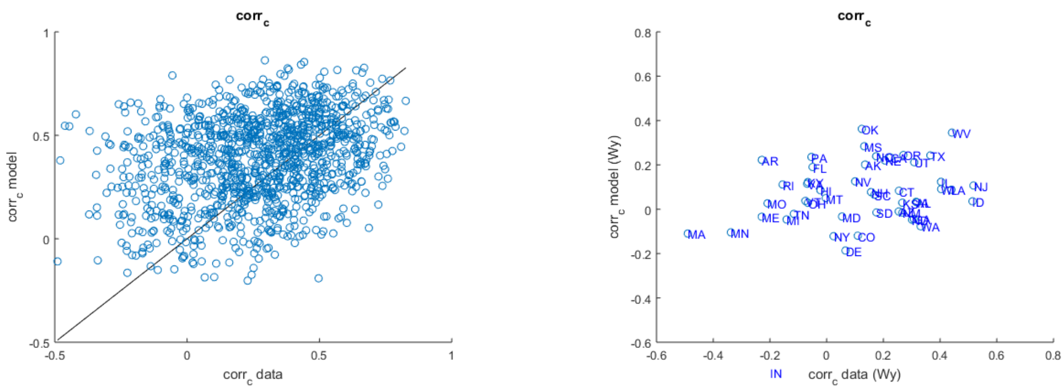
Lastly, we regress the estimated β_{ij} on geographic distance. Results reported in table A.3 suggest that the gravity model of consumption risk sharing remains robust under this alternative measure of output shocks.

Table A.3: Gravity Model of Risk Sharing – Alternative β and Alternative Distance

Dep. Var.: $\hat{\beta}_{ij}$	A. CPI by Hazell et. al.			B. Consumption from BEA		
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(d_{ij})$	0.119 ***	0.123 ***	0.155 ***	0.154 ***	0.158 ***	0.192 ***
	(0.017)	(0.017)	(0.022)	(0.010)	(0.010)	(0.012)
$\log(\bar{y}_1 \cdot \bar{y}_2)$		-0.035	-0.160 **		-0.089 ***	-0.115 ***
		(0.064)	(0.074)		(0.032)	(0.037)
$\log(\sigma(y_1) \cdot \sigma(y_2))$			0.152 ***			0.023
			(0.055)			(0.023)
Border			0.055			0.114 ***
			(0.056)			(0.023)
$\log(\bar{N}_1 \cdot \bar{N}_2)$			0.024 ***			0.029 ***
			(0.013)			(0.005)
Obs.	528	528	528	1225	1225	1225
R^2	0.077	0.077	0.102	0.163	0.169	0.194

Robust standard errors in parentheses. * significant at 10%, *** significant at 1%. The dependent variable in panel A is the adjusted risk sharing coefficient estimated using the real consumption and output data over 1977 – 2019 based on equation A1 and A2. $\log d_{ij}$ denotes the geographic distance between state i and j in logarithms. y_i and N_i denote real output per capita and population in logarithms of state i. $\bar{x}$ and $\sigma(x)$ represent the mean and volatility of a variable x over the sample period. Panel B uses the CFS-based geographic distance as an independent variable.

Figure A.3: Model Fit (II)



Note: This figure plots the relationship between model-implied and actual bilateral consumption correlations. Empirical moments are on the horizontal axis, and theoretical moments are on the vertical axis. The left diagram covers all the state pairs, the right covers the pairs formed by Wyoming as an example.

Table A.4: Estimated Frictions by State

	Trade Cost		Migration Cost		Financial Cost	
State	Outbound	Inbound	Outbound	Inbound	Outbound	Inbound
AL	0.975	1.476	1.035	1.117	0.493	0.592
AK	3.136	3.643	0.888	1.146	30.850	54.888

AZ	1.561	1.410	0.996	0.974	1.403	1.281
AR	1.007	2.296	1.002	1.115	1.562	0.754
CA	1.845	0.452	1.018	0.858	0.930	0.568
CO	1.406	1.520	0.934	0.966	1.379	1.864
CT	1.478	1.513	1.033	1.165	5.474	3.356
DE	1.536	2.822	1.069	1.175	80.416	72.842
FL	1.731	0.994	1.007	0.821	1.277	7.177
GA	1.057	1.113	0.970	0.973	1.292	1.393
HI	2.710	4.099	0.980	1.086	6.792	9.723
ID	1.045	2.719	1.019	1.159	3.249	5.006
IL	1.111	0.719	0.988	0.983	0.750	0.672
IN	0.917	1.042	0.999	1.044	3.381	2.784
IA	0.646	1.952	1.005	1.080	7.757	4.730
KS	0.702	2.099	0.978	1.060	3.390	2.600
KY	0.884	1.483	1.000	1.074	7.201	6.939
LA	1.151	1.729	1.030	1.105	2.384	3.223
ME	1.128	2.384	1.019	1.181	0.002	2.119
MD	1.766	1.660	1.029	1.058	9.218	3.651
MA	1.374	1.200	1.005	1.064	3.732	3.272
MI	0.938	1.189	1.030	1.038	2.645	4.517
MN	1.150	1.555	1.025	1.076	1.414	0.780
MS	0.865	2.047	1.014	1.153	2.014	6.122
MO	0.921	1.101	1.008	1.032	1.119	0.827
MT	1.291	2.440	0.975	1.152	0.022	0.201
NE	1.082	1.695	1.025	1.167	14.183	14.576
NV	1.319	2.052	0.980	1.086	1.060	1.493
NH	1.522	2.535	1.013	1.193	1.580	3.732
NJ	1.012	1.104	1.018	1.068	0.899	0.883
NM	2.197	2.103	0.998	1.128	8.109	14.685
NY	2.122	0.673	1.074	0.977	8.658	7.305
NC	0.901	1.339	1.018	0.957	0.646	0.924
ND	0.910	3.245	0.984	1.177	0.735	5.364
OH	0.943	0.887	1.030	1.027	0.708	0.607
OK	1.077	1.913	1.036	1.113	1.754	0.808
OR	1.083	1.585	1.027	1.128	3.052	3.060
PA	1.070	0.762	1.021	1.032	0.216	0.308
RI	1.081	3.156	1.068	1.213	0.690	1.087
SC	0.983	1.334	1.003	1.034	0.283	0.633
SD	0.909	3.413	0.997	1.162	11.196	11.012
TN	0.884	0.942	0.978	0.995	1.836	2.071
TX	1.236	0.690	0.999	0.849	1.249	1.208
UT	0.951	1.873	1.013	1.125	2.752	3.114
VT	1.082	4.098	1.035	1.214	0.023	0.374

VA	1.252	1.335	0.997	0.976	2.416	2.006
WA	0.954	1.330	1.018	1.006	1.188	1.222
WV	1.070	2.900	1.084	1.201	0.308	14.961
WI	1.166	0.957	1.037	1.082	0.926	0.692
WY	1.490	3.177	0.932	1.157	0.018	0.566

This table presents the normalized trade, migration, and financial costs averaged across state pairs for each state. Step 1, we calculate both inbound and outbound frictions averaged across $\mathcal{I}-1$ pairs a state i forms: ($x_i^{ex} = \text{mean}(x_{ij}),\ x_i^{in} = \text{mean}(x_{ji}), j \in [1, \mathcal{I}-\infty], x \in \{\tau, d, f\}$). Step 2, we normalize the average friction of Georgia and Ohio, the median states in terms of output per capita, to 1 in each channel: $x_{\text{GA,OH}}^{ex} = x_{\text{GA,OH}}^{in} = 1$. We report the ratio of state-level frictions from step 2 to the median states' in the table for cross-state comparison.

Table A.5: Counterfactual Consumption Relative to Benchmark

	Equilibrium Level $\bar{c}$		Volatility σ_c		
State	No τ	No d	No τ	No d	No f
Alabama	1.058	0.958	1.015	0.974	1.007
Alaska	1.298	0.955	0.908	0.969	0.981
Arizona	1.072	0.985	0.999	0.993	1.000
Arkansas	1.161	0.981	1.068	1.015	1.000
California	1.033	1.044	0.987	1.047	1.018
Colorado	1.067	0.978	1.036	1.009	1.049
Connecticut	1.092	0.998	0.939	1.006	0.993
Delaware	1.202	0.967	0.816	0.970	0.998
Florida	0.979	1.032	0.998	0.995	1.003
Georgia	1.026	0.983	0.966	0.971	1.003
Hawaii	1.094	0.977	0.953	0.979	1.000
Idaho	1.200	0.931	1.036	0.987	1.002
Illinois	1.009	0.978	0.972	0.994	1.002
Indiana	1.050	0.943	0.970	0.982	0.958
Iowa	1.064	0.947	0.879	0.967	1.010
Kansas	1.059	0.962	0.959	0.963	0.986
Kentucky	1.051	0.948	0.966	0.983	0.998
Louisiana	1.075	0.968	0.897	1.002	0.991
Maine	1.165	0.939	1.156	0.971	1.000
Maryland	1.070	0.974	1.003	0.990	1.001
Massachusetts	1.036	0.980	0.958	0.988	1.004
Michigan	1.021	0.993	0.958	0.999	1.005
Minnesota	1.082	0.972	0.997	0.966	1.006
Mississippi	1.127	0.954	1.033	0.990	0.991
Missouri	1.071	0.974	1.022	0.992	0.990
Montana	1.213	0.906	1.112	0.985	1.000
Nebraska	1.136	0.957	0.910	1.017	0.965
Nevada	1.097	0.968	0.979	0.985	1.000

New Hampshire	1.250	0.983	1.106	0.992	1.000
New Jersey	1.002	0.976	0.946	0.990	1.001
New Mexico	1.221	0.988	0.969	1.018	0.996
New York	1.027	1.018	0.956	1.038	1.000
North Carolina	1.024	0.989	0.975	1.004	0.969
North Dakota	1.263	0.919	1.041	1.032	1.000
Ohio	1.014	0.965	0.957	1.010	1.010
Oklahoma	1.080	0.964	0.997	0.984	0.981
Oregon	1.070	0.952	0.982	0.977	0.959
Pennsylvania	1.001	0.974	1.012	0.987	1.000
Rhode Island	1.197	0.946	1.117	0.984	1.007
South Carolina	1.091	0.959	1.080	0.965	1.003
South Dakota	1.245	0.903	0.901	0.928	0.951
Tennessee	1.075	0.955	0.999	0.981	1.000
Texas	0.964	0.993	0.932	1.031	1.032
Utah	1.135	0.962	0.971	0.979	0.995
Vermont	1.329	0.909	1.193	0.985	1.000
Virginia	1.001	0.979	0.999	0.994	1.000
Washington	1.033	0.989	0.923	1.005	1.001
West Virginia	1.093	0.941	1.070	1.001	1.004
Wisconsin	1.072	0.959	1.030	0.983	0.998
Wyoming	1.356	0.927	1.018	0.962	1.000
Mean	1.103	0.966	0.993	0.990	0.997
Median	1.073	0.968	0.984	0.988	0.999

This table presents each state's median counterfactual steady-state level and volatility of consumption across its state pairs, as a ratio to the values in original case with frictions calibrated to the data. Counterfactual scenarios include the cases absent bilateral trade costs (τ), migration costs (d), and financial frictions (f).

B Data

B.1 State-level output, consumption, and price

The US Bureau of Economic Analysis (BEA) reports state-level output, consumption, and price data in the Regional Economic Accounts (REA). Real GDP by state (GSP) data are available since 1977, with data from 1977-1997 reported in the Standard Industrial Classification (SIC) and those from 1997-2019 in the North American Industry Classification (NAICS). To address this discontinuity in coding, we first calculate the annual growth rate based on the SIC-based real GSP, and then reconstruct the time series of real GSP from 1977 to 1997 using this annual growth rate and the value of the NAICS-based real GSP in 1997.

The nominal consumption data from the BEA are only available after 1997, which is not ideal for our risk-sharing analysis over a long horizon. Therefore, we follow Asdrubali et al. (1996)'s method of constructing state-level private consumption by rescaling state-level retail sales by the country-level ratio of private consumption to retail sales, both obtained from the BEA. To convert nominal to real consumption, we use the state-level inflation series constructed by Nakamura and Steinsson (2014) over the period from 1966 to 2008. They obtain the inflation series from 1966 to 1995 from Del Negro (1998), who constructs the series using a combination of BLS regional inflation data and cost-of-living estimates from the American Chamber of Commerce Realtors Association (ACCRA). For the estimates between 1995 and 2008, they multiply a population-weighted average of cost-of-living indices from the ACCRA across states with the US aggregate CPI. After 2008, we use the Regional Price Parities (RPP) from the BEA that measure price differences within the United States. RPP is a weighted average of the price level of goods and services for the average consumer in one geographic region compared to all other regions in the US. We merge these two series to construct a state-level CPI index for 1966-2019, using which we deflate the nominal consumption data to calculate real consumption at the state level.

We conduct sensitivity analyses using alternative data sources to verify the robustness of the gravity model. Table A.2 Panel A uses the state-level inflation rates from Hazell et al. (2022) who construct CPI with micro data gathered by the BLS from 1978 to 2017. Panel B uses only the recent BEA data of consumption expenditure and real GSP between 1997 and 2018. The gravity model of consumption risk sharing remains robust under these alternative data sources.

B.2 Bilateral trade and migration flows

The Commodity Flow Survey (CFS) is conducted every five years by the U.S. Census Bureau in partnership with the U.S. Department of Transportation. The survey provides detailed information on the U.S. commodity flows, including the type of commodities shipped, origin and destination, value and weight, and mode of transport. There are six waves so far (1993, 1997, 2002, 2007, 2012, 2017), which allow us to map dynamic spatial patterns of commodity flows in the US.

State-to-state migration data are based on year-to-year address changes reported on individual income tax returns filed with the Internal Revenue Service (IRS). Specifically, we use the reported number of returns filed every year to track migration patterns across states. The data are available for filing years 1991 through 2019.

B.3 State-level productivity

In this multi-region framework, we estimate the total factor productivity (TFP) for 50 states in the US. In particular, we examine the Solow residual from

$$\log(A_{i,t}) = \log(Y_{i,t}) - \alpha \log(K_{i,t}) - (1-\alpha) \log(L_{i,t}), \tag{A3}$$

where $Y_{i,t}$, $K_{i,t}$, and $L_{i,t}$ are output, capital, and labor in state i at time t respectively, while α denotes capital share in production. We use the Bureau of Economic Analysis (BEA) data over the period 1977-2019 for the estimation. The BEA reports state-level gross domestic product and employment in the Regional Economic Accounts. It also provides the national and sectoral capital data in the Fixed Assets Accounts.

We construct the estimates for state-level capital stock using the methodology developed by Garofalo and Yamarik (2002). Namely, we apportion the national capital stock, measured as the net stock of total private fixed assets net of residential fixed assets, to the states using sector-level income data. For each two-digit NAICS industry s, we apportion the national capital stock based on the relative income generated within each state as follows:

$$K^s_{i,t} = \left(\frac{Y^s_{i,t}}{Y^s_{US,t}}\right) K^s_{US,t}, \tag{A4}$$

where $K^s_{i,t}$ ($Y^s_{i,t}$) refers to capital (output) of industry s in state i at time t, while $K^s_{US,t}$ and $Y^s_{US,t}$ represent the country-level variables. Each state's capital stock estimate, $K_{i,t}$, is then the sum of sectoral-level capital stock:

$$K_{i,t} = \sum_{s=1}^{K} K^s_{i,t}. \tag{A5}$$

Lastly, We estimate $1-\alpha$ in equation A3 to be 0.59 by dividing the labor earnings by the economic output based on the BEA data over the sample period.[9] After obtaining the values of all the elements that appear in equation A3, we calculate the state-level TFP with which we subsequently estimate the joint productivity process across states.

C Portfolio Choice in Trilateral Framework

In this section I describe and solve the portfolio choice problem introduced in the theory section within a framework with three economies numbered $i = 1, 2, 3$. In particular, region 3 can be regarded as the rest of the economy from the perspective of the region-pair formed by regions 1 and 2. Each economy's financial asset, which can be traded in an integrated financial market, is its claims to capital income net of investment expenditure. Nevertheless, there are bilateral financial frictions modeled as capital gain taxes f_{ij} on returns R_i when j holds assets from i. These second-order frictions appear

[9] The BEA provides the labor earning data (SAINC5). The earning consists of compensation of employees and proprietors' income with inventory valuation adjustment and capital consumption adjustment.

in the Euler equations of the three economies respectively given by

$$\begin{aligned} E_t[\tfrac{U'(c_{1,t+1})}{P_{1,t+1}}R_{1,t+1}] &= E_t[\tfrac{U'(c_{1,t+1})}{P_{1,t+1}}e^{-f_{21}}R_{2,t+1}] &= E_t[\tfrac{U'(c_{1,t+1})}{P_{1,t+1}}e^{-f_{31}}R_{3,t+1}], \\ E_t[\tfrac{U'(c_{2,t+1})}{P_{2,t+1}}R_{2,t+1}] &= E_t[\tfrac{U'(c_{2,t+1})}{P_{2,t+1}}e^{-f_{12}}R_{1,t+1}] &= E_t[\tfrac{U'(c_{2,t+1})}{P_{2,t+1}}e^{-f_{32}}R_{3,t+1}], \\ E_t[\tfrac{U'(c_{3,t+1})}{P_{3,t+1}}R_{3,t+1}] &= E_t[\tfrac{U'(c_{3,t+1})}{P_{3,t+1}}e^{-f_{13}}R_{1,t+1}] &= E_t[\tfrac{U'(c_{3,t+1})}{P_{3,t+1}}e^{-f_{23}}R_{2,t+1}]. \end{aligned} \tag{A6}$$

In the next step we derive portfolios with Devereux and Sutherland (2011)'s method by evaluating the second-order approximation of these Euler equations. First we assume assets from economy 3 to be a numeraire asset and denote the vector of excess returns to the other assets as R_x:

$$\hat{R}'_{x,t} = [\hat{R}_{1,t} - \hat{R}_{3,t}, \hat{R}_{2,t} - \hat{R}_{3,t}], \tag{A7}$$

where $\hat{y}_t$ represents the log-deviation of any variable y from its steady state at t. Next we evaluate the second-order Taylor expansion of the Euler equations as

$$\begin{aligned} E_t[\hat{R}_{x,t+1} + \tfrac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{1,t+1} + \hat{P}_{1,t+1})\hat{R}_{x,t+1}] &= -\tfrac{1}{2}\begin{bmatrix} f_{31} \\ f_{31} - f_{21} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[\hat{R}_{x,t+1} + \tfrac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{2,t+1} + \hat{P}_{2,t+1})\hat{R}_{x,t+1}] &= -\tfrac{1}{2}\begin{bmatrix} f_{32} - f_{12} \\ f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[\hat{R}_{x,t+1} + \tfrac{1}{2}\hat{R}^2_{x,t+1} - (\sigma\hat{c}_{3,t+1} + \hat{P}_{3,t+1})\hat{R}_{x,t+1}] &= -\tfrac{1}{2}\begin{bmatrix} -f_{13} \\ -f_{23} \end{bmatrix} + \mathcal{O}(\epsilon^3). \end{aligned} \tag{A8}$$

where $\hat{R}^{2'}_{x,t+1}$ denotes differences in squared changes of returns

$$\hat{R}^{2'}_{x,t+1} = [\hat{R}^2_{1,t+1} - \hat{R}^2_{3,t+1}, \hat{R}^2_{2,t+1} - \hat{R}^2_{3,t+1}]. \tag{A9}$$

On the right-hand side of equations A8 are vectors of financial frictions each country incurs when holding assets from economies 1 and 2 relative to the frictions associated with its holding assets from economy 3. Plus, the last term $\mathcal{O}(\epsilon^3)$ captures all terms of order higher than two. Taking the difference among equations A8 yields

$$\begin{aligned} E_t[(\hat{c}_{12,t+1} + \tfrac{\hat{P}_{12,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \tfrac{1}{2\sigma}\begin{bmatrix} f_{31} - f_{32} + f_{12} \\ f_{31} - f_{21} - f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[(\hat{c}_{13,t+1} + \tfrac{\hat{P}_{13,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \tfrac{1}{2\sigma}\begin{bmatrix} f_{13} + f_{31} \\ f_{31} - f_{21} + f_{23} \end{bmatrix} + \mathcal{O}(\epsilon^3), \\ E_t[(\hat{c}_{23,t+1} + \tfrac{\hat{P}_{23,t+1}}{\sigma})\hat{R}_{x,t+1}] &= \tfrac{1}{2\sigma}\begin{bmatrix} f_{32} - f_{12} + f_{13} \\ f_{23} + f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3), \end{aligned} \tag{A10}$$

where $c_{ij,t} = \frac{c_{i,t}}{c_{j,t}}$ and $P_{ij,t} = \frac{P_{i,t}}{P_{j,t}}$ denote cross-region consumption and price ratios of i to j, which constitute a vector of price-adjusted consumption differential defined as

$$\frac{\hat{cp}'_t}{\sigma} = [\hat{c}_{12,t} + \frac{\hat{P}_{12,t}}{\sigma}, \hat{c}_{13,t} + \frac{\hat{P}_{13,t}}{\sigma}, \hat{c}_{23,t} + \frac{\hat{P}_{23,t}}{\sigma}]. \tag{A11}$$

Equations A10 can therefore be re-written in the vector form as

$$E_t[\hat{cp}_t \hat{R}'_{x,t+1}] = \frac{\mathcal{F}}{2} \equiv \frac{1}{2} \begin{bmatrix} f_{31} - f_{32} + f_{12} & f_{31} - f_{21} - f_{32} \\ f_{13} + f_{31} & f_{31} - f_{21} + f_{23} \\ f_{32} - f_{12} + f_{13} & f_{23} + f_{32} \end{bmatrix} + \mathcal{O}(\epsilon^3). \tag{A12}$$

On the left hand side of this portfolio determination equation are two components: inflation-adjusted consumption differential $\hat{cp}$ and excess financial returns $\hat{R}_x$. Both components can be expressed in terms of region-specific innovations

$$\epsilon'_t = [\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t}], \tag{A13}$$

whose coefficients, as a function of portfolio choice, need to satisfy equation A12 in the equilibrium of the model. Let $\alpha_{i,j}$ represent j's holding of asset i, then the unknown portfolio matrix scaled by the discount factor β and the region's steady-state output $\bar{Y}$ to be solved in this three-economy framework is

$$\tilde{\alpha} = \frac{1}{\beta \bar{Y}} \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix}, \tag{A14}$$

while the remaining holdings $\alpha_{3,j}$ and $\alpha_{i,3}$ can be recovered from each region's budget constraint and asset market clearing condition respectively. Given the portfolio arrangement, excess portfolio return is defined as

$$\xi_t = \tilde{\alpha}' \hat{R}_{x,t}. \tag{A15}$$

Region-specific productivity shocks ϵ_t affect the two components in equation A12 both directly and indirectly through ξ_t:

$$\hat{cp}_{t+1} = D_1 \xi_{t+1} + D_2 \epsilon_{t+1} + D_3 z_{t+1} + \mathcal{O}(\epsilon^2), \tag{A16}$$

$$\hat{R}_{x,t+1} = R_1 \xi_{t+1} + R_2 \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \tag{A17}$$

where R_1, R_2, D_1, D_2, D_3 are the coefficient matrices extracted from the first-order conditions of the model. R_1 and D_1 capture the response of the two components (consumption differential and excess asset returns) to excess portfolio returns; R_2 and D_2 capture their response to productivity shocks; and D_3 are their response to other state variables in the model summarized by z. In addition, using $\xi_{t+1} = \tilde{\alpha}' \hat{R}_{x,t+1}$ allows us to express ξ_{t+1}, $\hat{cp}_{t+1}$, and $\hat{R}_{x,t+1}$ in terms of ϵ_{t+1} only:

$$\xi_{t+1} = \tilde{H} \epsilon_{t+1}, \quad \textit{where} \quad \tilde{H} = \frac{\tilde{\alpha}' R_2}{1 - \tilde{\alpha}' R_1}; \tag{A18}$$

$$\hat{cp}_{t+1} = \tilde{D} \epsilon_{t+1} + D_3 z_{t+1} + \mathcal{O}(\epsilon^2), \quad \textit{where} \quad \tilde{D} = D_1 \tilde{H} + D_2. \tag{A19}$$

$$\hat{R}_{x,t+1} = \tilde{R} \epsilon_{t+1} + \mathcal{O}(\epsilon^2), \quad \textit{where} \quad \tilde{R} = R_1 \tilde{H} + R_2. \tag{A20}$$

Now that we have examined the two components in equation A12 separately as functions of innovations ϵ_{t+1}, we can multiply them to evaluate the portfolio determination condition:

$$E_t[\hat{c}p_t \hat{R}'_{x,t+1}] = \tilde{D}\Sigma\tilde{R}' = \frac{\mathcal{F}}{2}. \tag{A21}$$

In terms of computation, we follow the steps below to numerically estimate bilateral financial frictions f_{ij}. First, we extract coefficient matrices R_1, R_2, D_1, D_2, and the response of the relative output differential $\hat{y}_{ij} = \hat{y}_i - \hat{y}_j$ to shocks from the first order conditions in the model. In particular, we take the first order derivative of output differential to productivity shocks

$$Dy = \frac{\partial y_{ij}}{\partial \epsilon}, \tag{A22}$$

where ϵ is the vector of productivity shocks defined in A13. We use the same method to capture the response of the relative consumption differential $\hat{c}_{ij} = \hat{c}_i - \hat{c}_j$ to shocks

$$Dc = \frac{\partial c_{ij}}{\partial \epsilon}, \tag{A23}$$

which based on equation A19 is influenced by portfolio choice $\tilde{\alpha}$ from A14 together with coefficient matrices R_1, R_2, D_1, D_2 calculated earlier. The coefficient of consumption risk sharing $\hat{\beta}_{ij}$ can therefore be approximated as

$$\hat{\beta}_{ij} = \frac{\partial c_{ij}}{\partial y_{ij}} = \frac{Dc}{Dy}. \tag{A24}$$

After we compute $\hat{\beta}_{ij}$ for each productivity shock following the steps above using the first-order dynamics of the model, we take the mean value of $\hat{\beta}_{ij}$ across shocks to get a state-pair's overall consumption risk sharing and compare it with the coefficient estimated from the empirical section which serves as a targeted moment for the state-pair under examination. We loop over different portfolios $\tilde{\alpha}$ until the model-predicted coefficient of risk sharing matches its empirical moment. After that, we plug the calibrated portfolio $\tilde{\alpha}$ in $\tilde{D}$ and $\tilde{R}$ of equation A21 to find matrix $\mathcal{F}$. Lastly, we recover bilateral financial frictions from this matrix of financial frictions based on equation A12.